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Thermodynamics and Fluid Mechanics 2

Fluids Topic 3: Lift and drag

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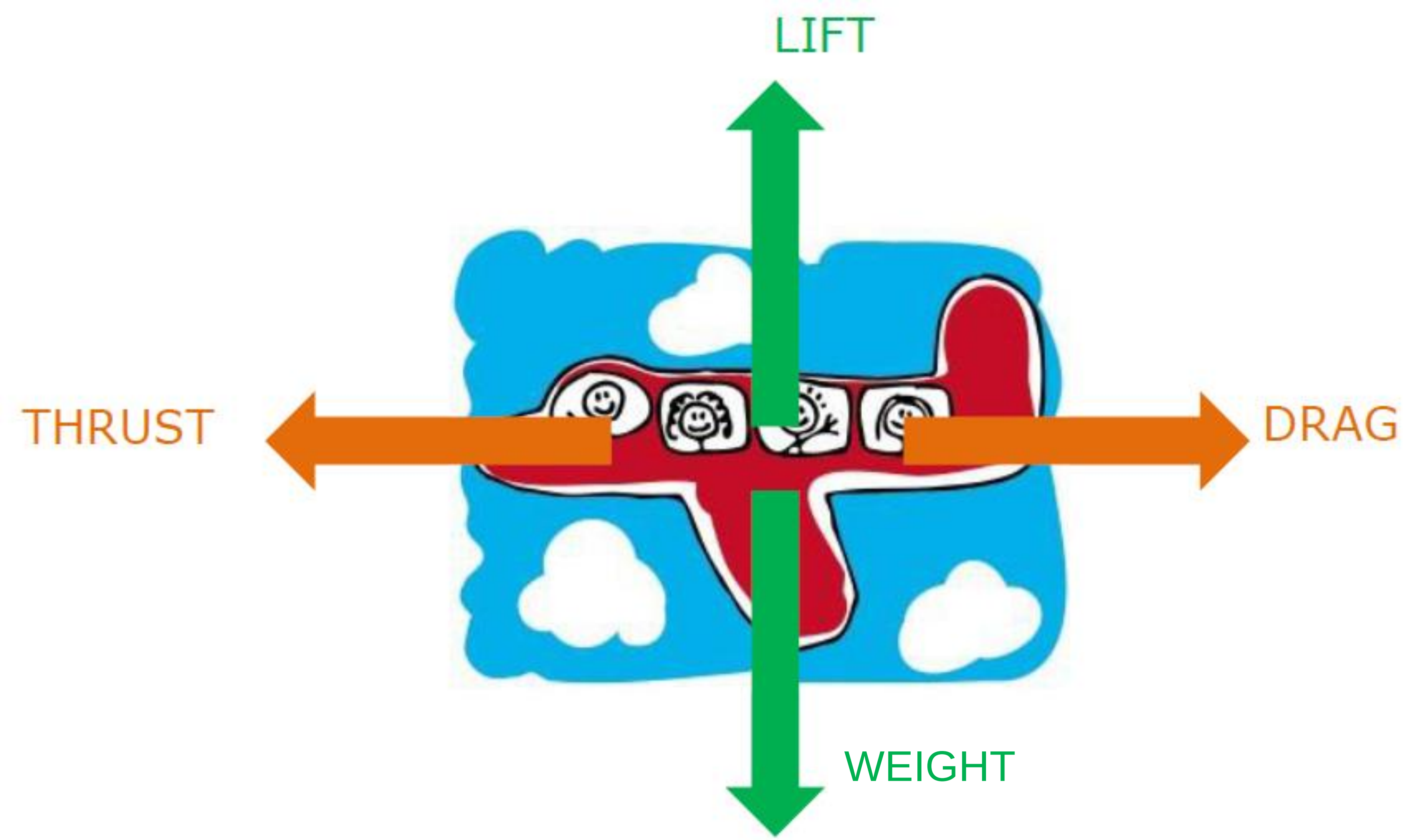
Topic 3 – Lift and drag



Topic 3 – Lift and drag

- Forces acting on a body immersed in a fluid stream
- Friction and pressure drag
- Drag on streamlined and bluff bodies
- Drag for flow past a cylinder, laminar/turbulent flow, flow separation
- Effect of surface roughness on flow past cylinders and spheres
- Drag over 2D bodies
- Drag over 3D bodies, impact of spanwise length
- Lift force and lift coefficient
- Lift and drag over airfoils and impact of flaps
- Flow separation and stall
- Lift and drag over rotating bodies

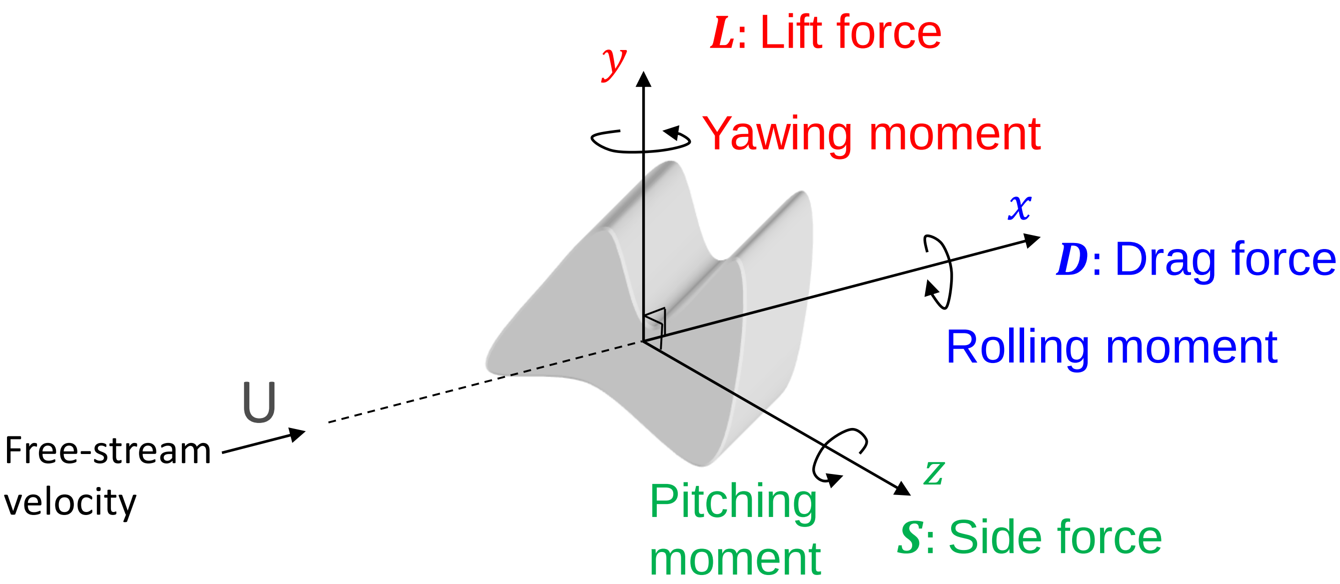
Topic 3 can be studied in F. White, Ch. 7

Learning outcomes:

- Recognise and sketch the different forces acting on a body
- Know how to relate drag and lift forces to drag and lift coefficients
- Know how to calculate the appropriate Reynolds number of the flow depending on the shape of the body
- Know how to calculate the drag coefficient based on the Reynolds number of the flow and shape of the body
- Distinguish the different flow regimes for flow past a cylinder
- Be able to calculate the drag force based on tabulated drag coefficients for 2D and 3D bodies
- Extract lift coefficients for airfoils from charts, depending on the angle of attack
- Evaluate the minimum speed to avoid airfoil stall

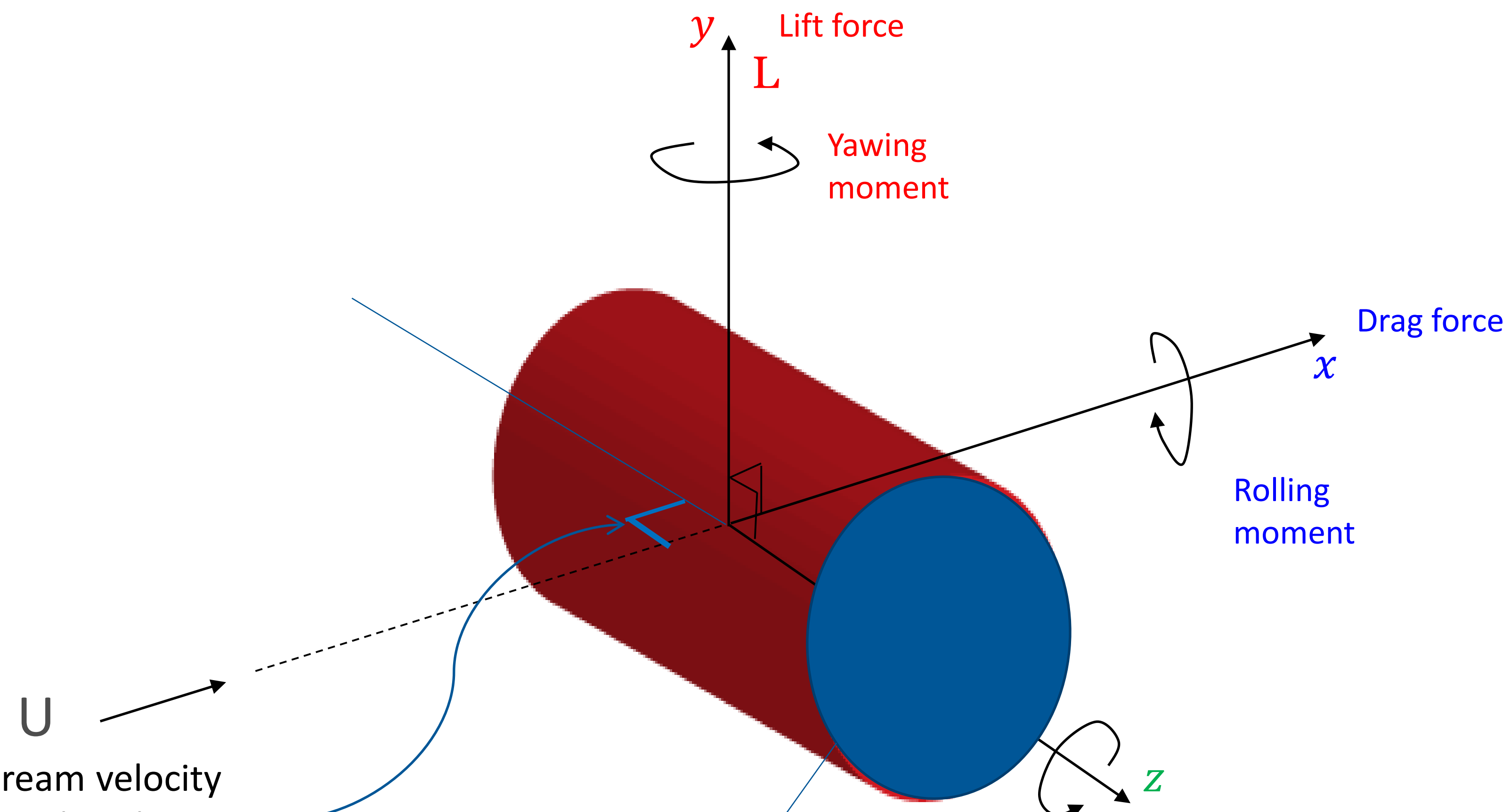
Forces on a body in a fluid stream

- Drag force: acting along the flow direction
- Lift force: acting perpendicular to the flow direction, usually taken vertical
- Side force: acting perpendicular to the flow direction, usually taken horizontal
- Rolling moment: moment about the drag axis
- Yawing moment: moment about the lift axis
- Pitching moment: moment about the side axis



Forces on a body in a fluid stream

Example: forces acting on a cylinder with free stream at 90 deg angle

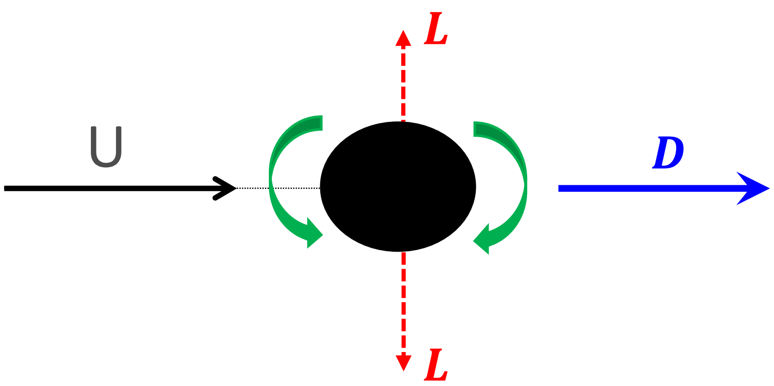


Free-stream velocity orthogonal to the cylinder axis

Look first at this plane

Forces on a body in a fluid stream

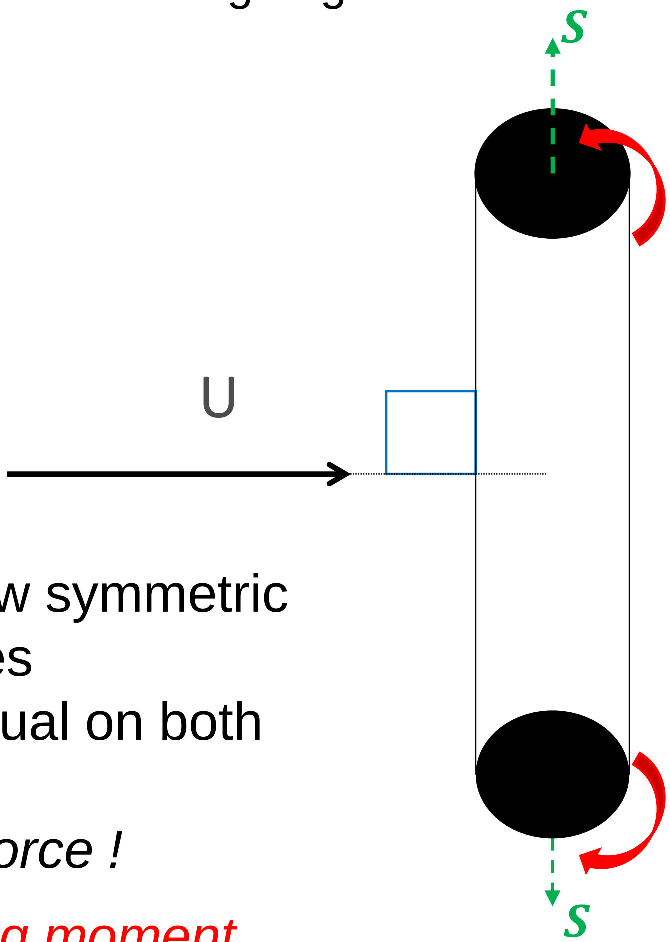
Example: forces acting on a cylinder with free stream at 90 deg angle



Shape & flow symmetric above/below
-> forces above/below equal
-> *No lift !*

-> *No pitching moment*

-> *No rolling moment*



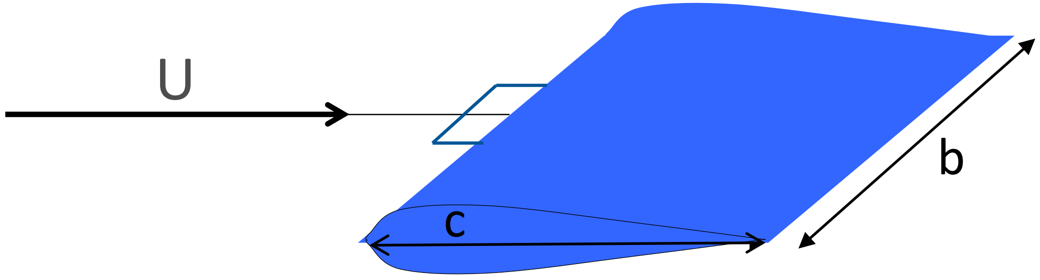
Shape & flow symmetric on both sides
-> forces equal on both sides/ends
-> *No side force !*

-> *No yawing moment*

A circular cylinder with axis orthogonal to the flow direction has only **drag!**
This applies in general to bodies of revolution, when the flow is perpendicular to their revolution axis.

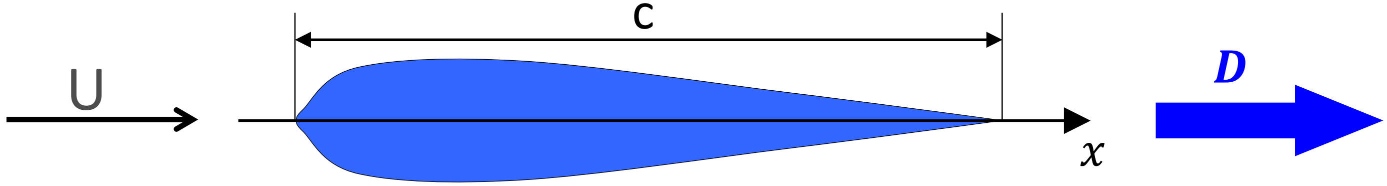
Forces on a body in a fluid stream

Example: symmetric airfoil

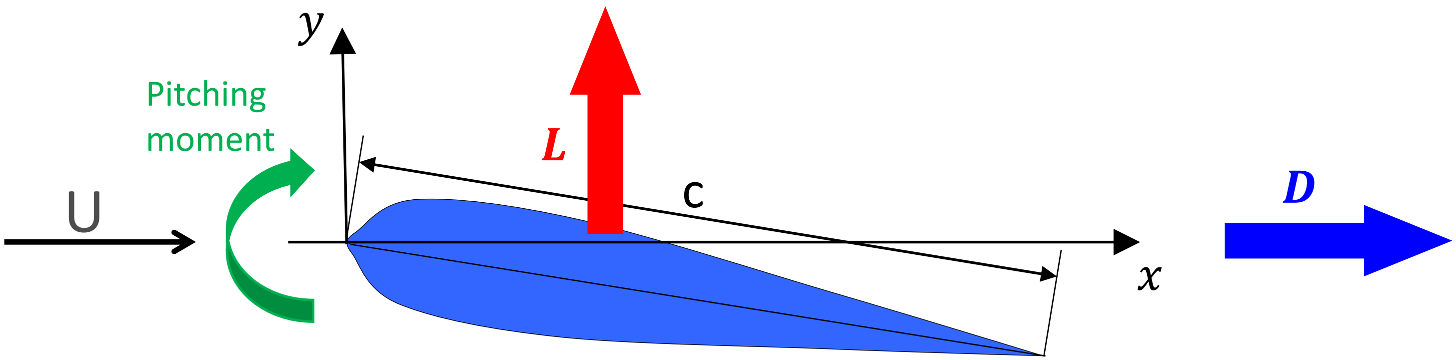


$if \;\; b \gg c$:
We can consider a 2D configuration

Free stream parallel to chord line: Only **drag**



Free stream not parallel to chord line: **drag, lift** and **pitching moment**



Importance of lift & drag in motorsport

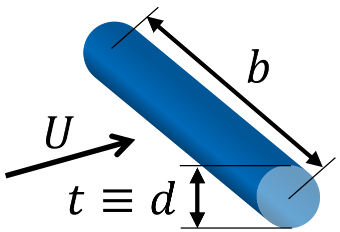
https://www.youtube.com/watch?v=OQN6S3RdV0A&t=45s

Drag coefficient

Drag force: $D = C_D\left(\frac{1}{2}\rho U^2\right)A$

The drag coefficient is obtained from experiments or simulations (from theory only for very few simple cases, e.g. flow over a flat plate), by measuring the drag force over the body, then:

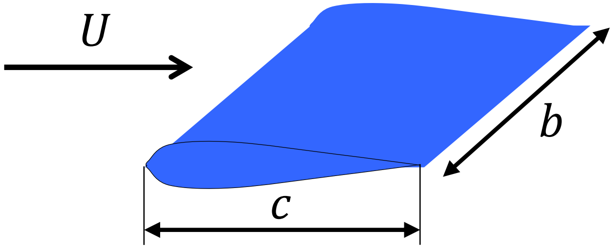
$$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$$



What to use for the area A?

- <u>Frontal area</u> for thick, stubby, *bluff* bodies (cylinders, spheres, cars, etc.): $A = tb$
- <u>Planform area</u> for flat, *streamlined* bodies (wings, air/hydro-foils): $A = cb$

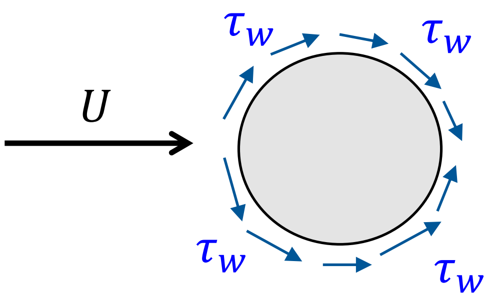
C_D is found on tables for different bodies, and the tables specify if the value is based on the frontal or planform area.

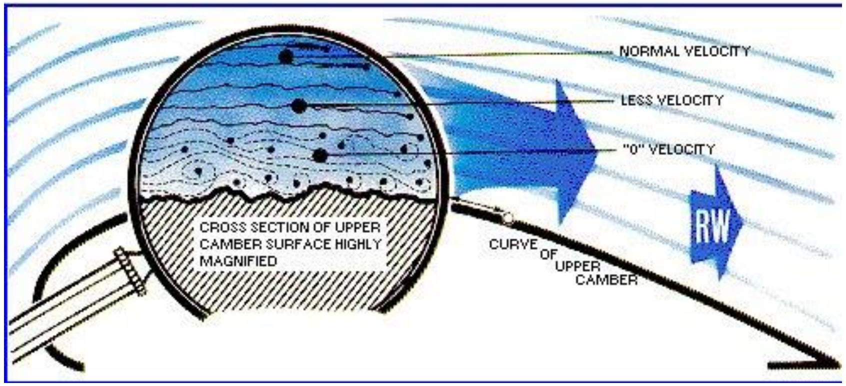


Friction drag and pressure drag

Friction drag

A body in a fluid stream is subject to friction drag, due to no-slip condition at the wall, and thus the wall shear generated by the fluid. We know this from T2.

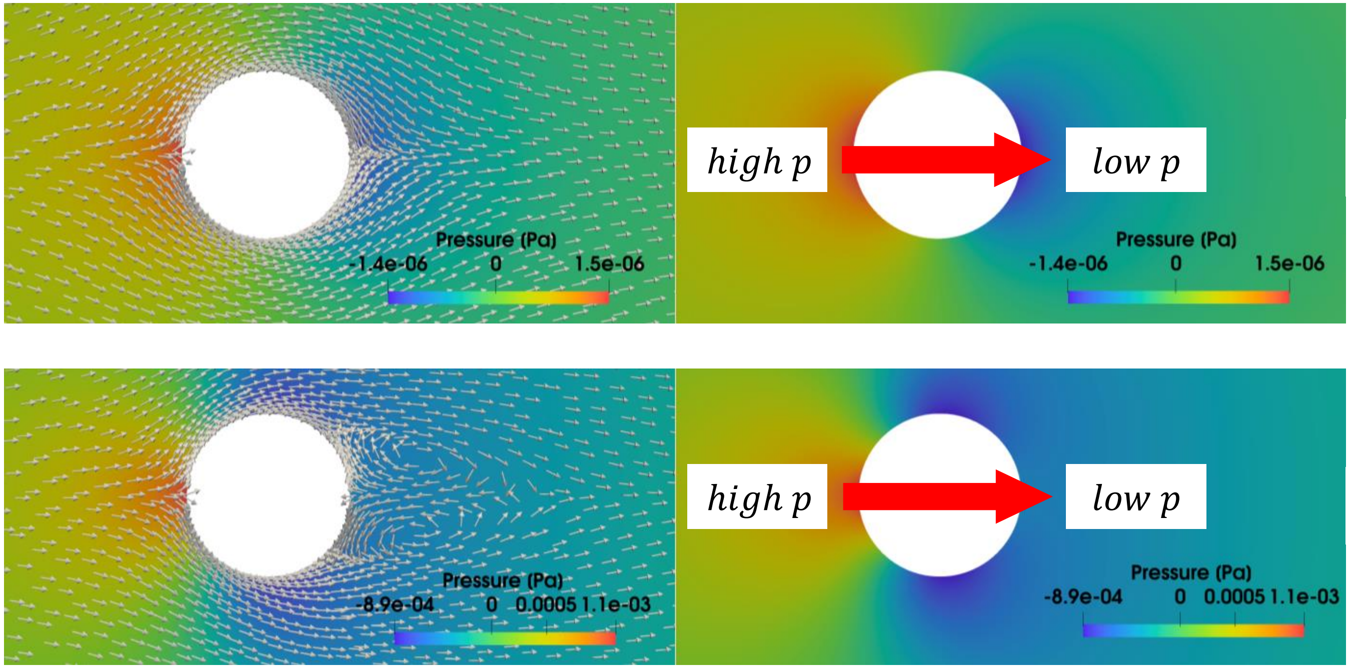




- Frictional drag increases with the planform area (projection of the area of the body parallel to the flow)
- Is larger for turbulent flows (because proportional to the velocity gradient at the wall)
- Is larger for a rough wall

Friction drag and pressure drag

Furthermore, at the end of T2 we saw that the pressure is not constant around a body, but it is larger upstream (stagnation point) and lower downstream.



$Re_D = 0.1$

$Re_D = 20$

This generates a force on the body, the **pressure drag**:

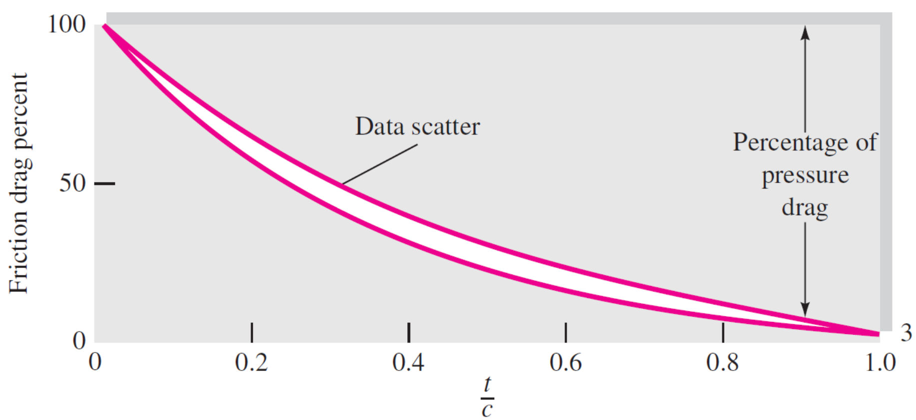
- Proportional to the frontal area of the body (and to the pressure difference)!
- Severely affected by flow separation, which makes the pressure in the wake much lower $\Longrightarrow$ to minimize pressure drag, we need to avoid flow separation!

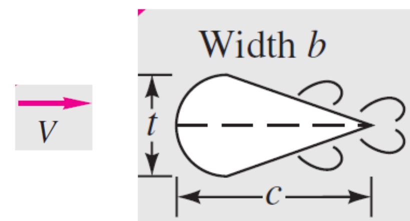
Friction drag and pressure drag

Therefore, the total drag force is the sum of two components:

$$D = D_{friction} + D_{pressure} = C_D\left(\frac{1}{2}\rho U^2\right)A$$

Where C_D is the sum of friction+pressure drag, $C_D = C_{D,friction} + C_{D,pressure}$, and tables usually provide the sum of the two. How much important is pressure drag?





In the figure:

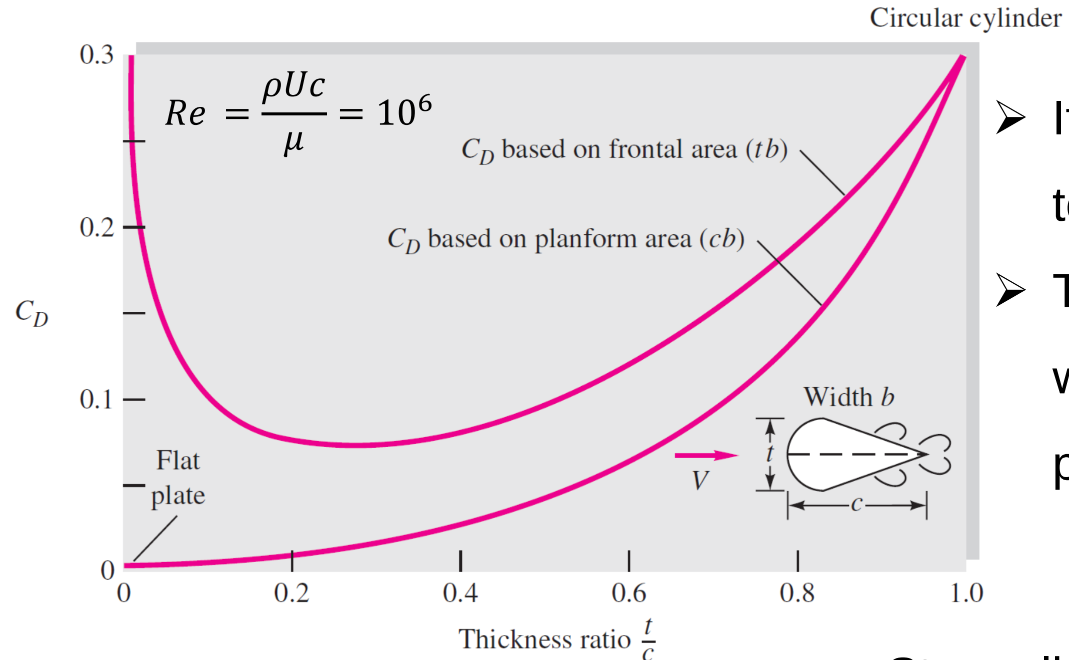
$$Re = \frac{\rho U c}{\mu} = 10^6$$

It depends on the shape of the body:

- Flat plate, $t/c \to 0$, therefore 100% of drag is due to friction
- Bluff bodies, e.g. a circular cylinder where $t/c = 1$, most of the drag is due to pressure (although this depends on Re)
- In between: the percentage of pressure drag increases with t

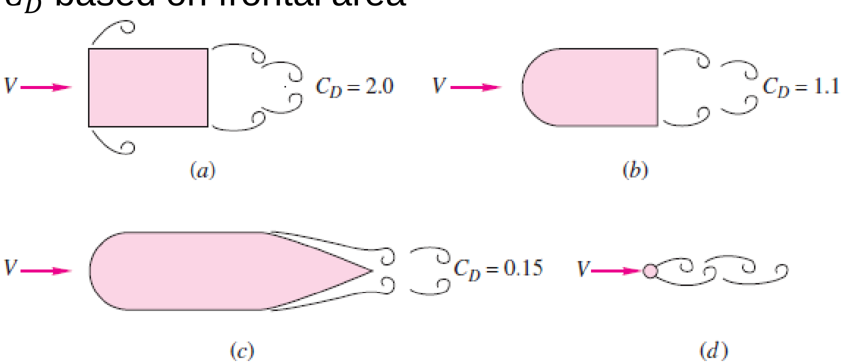
Drag coefficient – effect of the body shape

The value of the drag coefficient in general depends on: the shape of the body



- It increases when t/c increases, due to flow separation
- Tables/charts give C_D and specify whether it's based on the frontal or planform area

C_D based on frontal area



Streamlining the body helps reducing (pressure) drag:

(a) Rectangular cylinder: lots of separation.

(b) Same frontal area, but rounded nose.

(c) Same frontal area, but streamlined body.

(d) Circular cylinder with same C_D as (c)

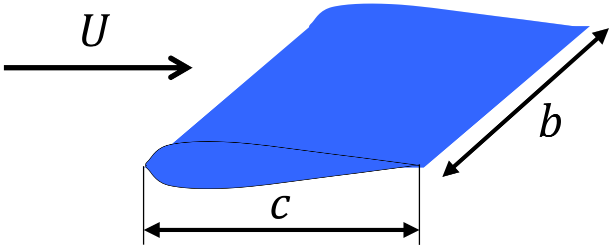
Drag coefficient – effect of Reynolds number

The value of the drag coefficient in general depends on: the Reynolds number

$$Re = \frac{\rho U \ell}{\mu}$$

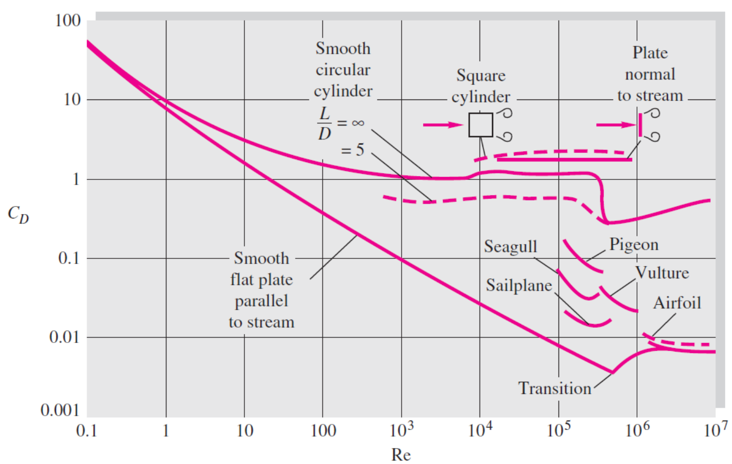
What to take as the length scale ℓ ?

- Bluff bodies – most of drag is due to pressure drag, which depends on the frontal area $\Longrightarrow$ use thickness t (d for cylinders)



- Streamlined bodies – most of drag is due to friction drag, which depends on planform area $\Longrightarrow$ use chord length c

Example: flow past a cylinder



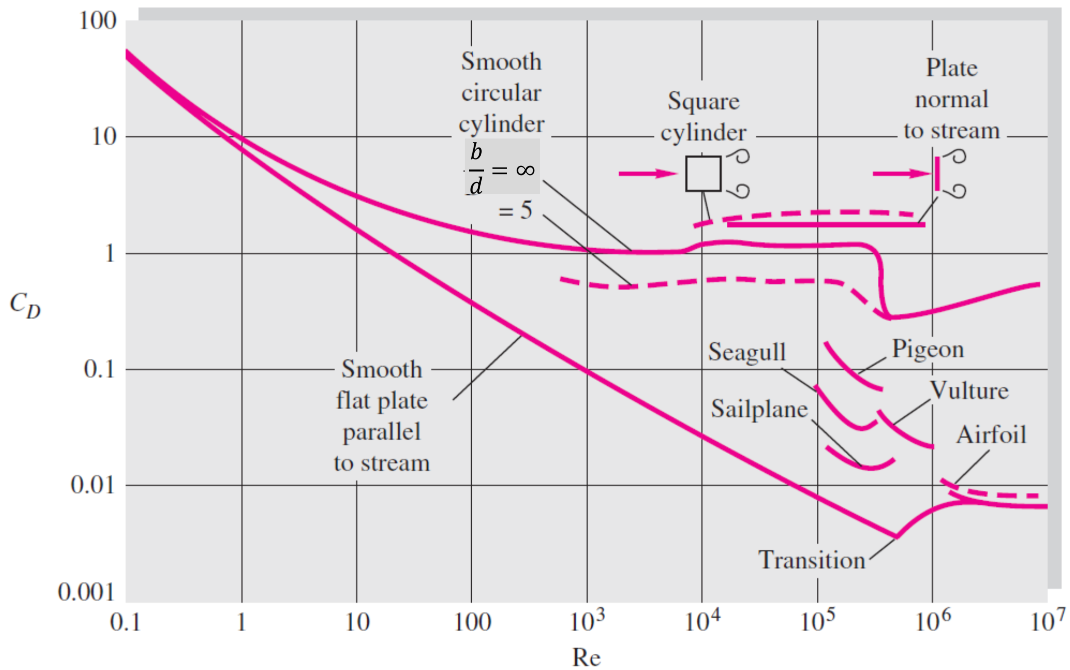
What is the drag force on the pole ($d = 1\ cm, b = 2\ m$) of a flag perpendicular to the wind direction ($U = 15\ m/s$)? Air: ρ=1.2 kg/m^3, μ=1.8·10^{-5} Pa·s

$$Re = \frac{\rho U d}{\mu} = 10^4 \Longrightarrow C_D \approx 1.2$$

$$D = C_D \left(\frac{1}{2}\rho U^2\right) bd = 3.24\ N$$

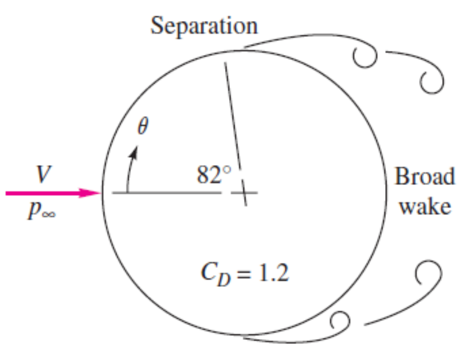
Drag coefficient – flow past a cylinder

Example: flow past a cylinder $Re = \frac{\rho U d}{\mu}$

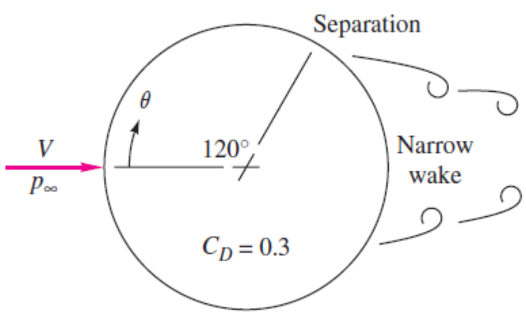


- $C_D \downarrow$ when $Re \uparrow$, down to an asymptotic value $C_D \approx 1.2$ for laminar flow
- When the flow becomes turbulent, C_D jumps to $C_D \approx 0.3$. **Why??** Turbulent flow resists better to pressure gradients, separation occurs at larger θ_s**,** and thus the wake region is smaller ⟹much less pressure drag!
- Note: bluff bodies (square cylinder, plate normal to stream), have $C_D \neq C_D(Re)$

Laminar flow separation

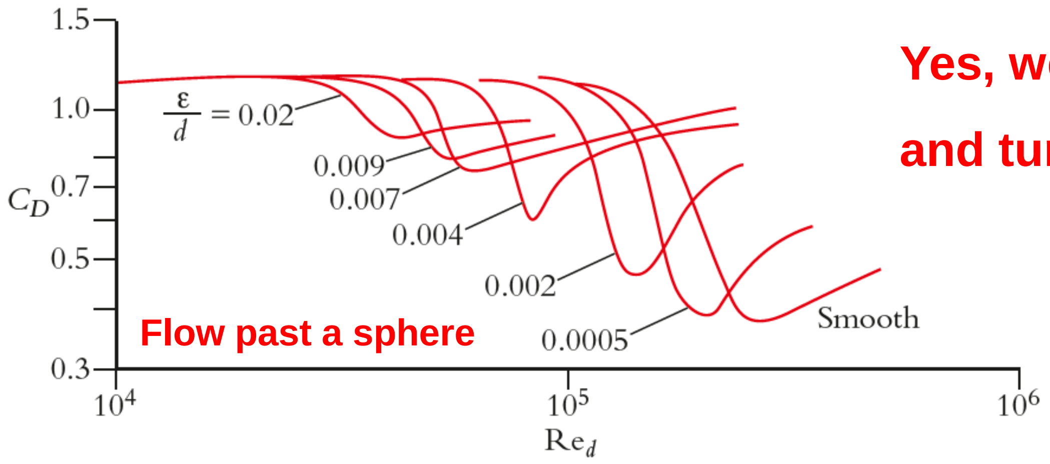


Turbulent flow separation



Drag coefficient – effect of surface roughness

So, wait a minute, you just said that C_D is much smaller if the flow is turbulent, can we make transition to turbulence happen earlier, that is, at lower Re?



Yes, we can make the surface rough, and turbulence will occur earlier!

But didn't we say in T2 (slide 43-44) that surface roughness increases C_D? Yes, roughness does increase friction drag, but it also decreases pressure drag (if it makes the flow turbulent). If pressure drag ≫ friction drag, then the overall effect of surface roughness is positive!

That's the reason why golf balls have dimples!

Drag coefficient – effect of surface roughness

Simulation of the flow past a dimpled golf ball:

https://www.youtube.com/watch?v=7W0P1Dk5-LE

Worked example 1

A golf ball has a diameter of 1.68 in (https://en.wikipedia.org/wiki/Golf_ball).

(a) What is the minimum ball speed at which the drag coefficient will be significantly affected by the dimples?

(b) I don't go to the gym since years. If I hit the ball, this won't go faster than 50 km/h. Does it make sense for me to use a golf ball with dimples?

(c) Tiger Woods can launch the ball at 100 mph. Where does this value sit at the Re-axis? Could Tiger Woods use a golf ball without dimples?

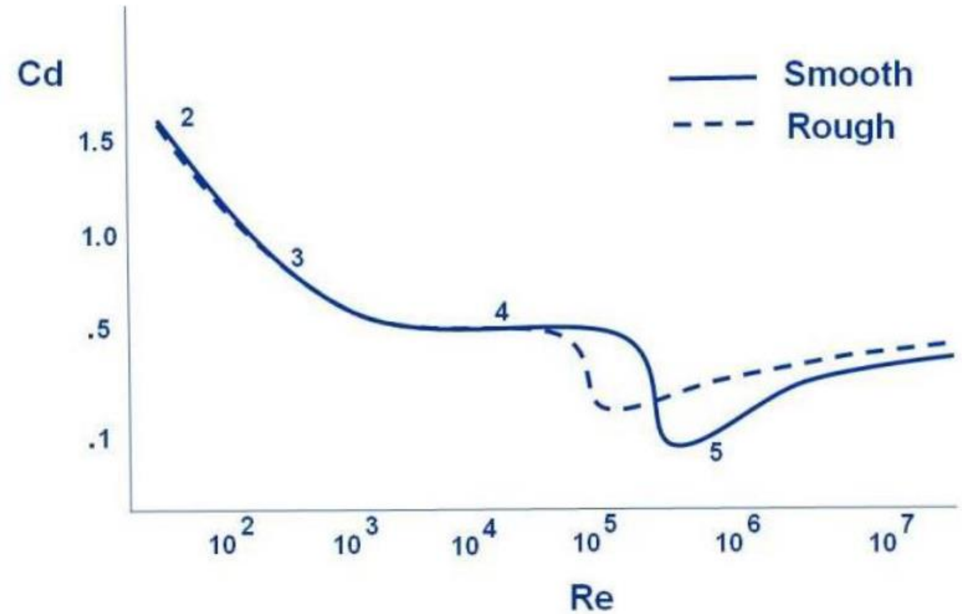


Chart of $C_D - Re$ for smooth and rough spheres

Worked example 1

Solution

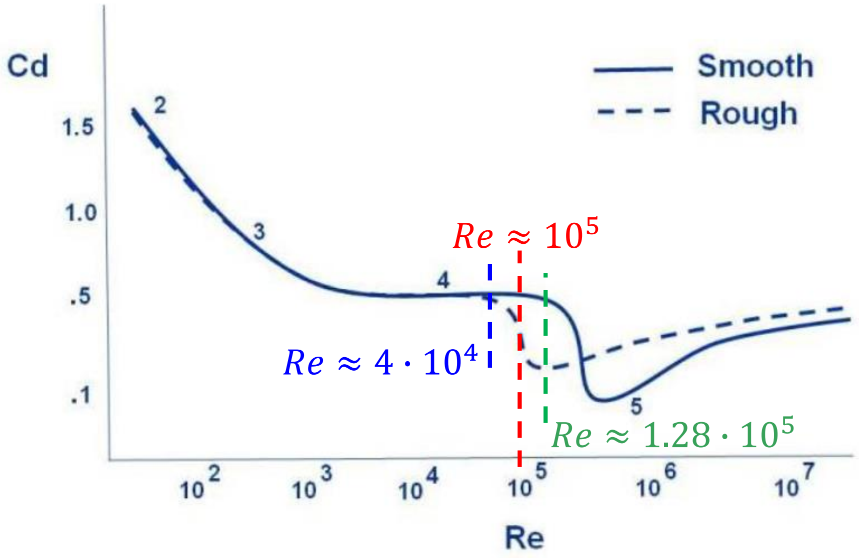
Air: ρ=1.2 kg/m³, m=1.8·10⁻⁵ Pa·s.

(a) From the chart, we see that we need $Re \approx 10^5$ for the dimples to have a significant impact on the drag coefficient. Therefore:

$$Re = \frac{\rho U d}{\mu} \Longrightarrow U = \frac{\mu Re}{\rho d} \approx 35\,\frac{m}{s} = 126\,\frac{km}{h}$$

(b) $U = 50\,\frac{km}{h} \approx 13.9\frac{m}{s} \Longrightarrow Re = \frac{\rho U d}{\mu} \approx 4 \cdot 10^4$

The dimples will not have a significant impact on my shot



(c)

$U = 100\,\frac{miles}{h} \approx 44.7\frac{m}{s} \Longrightarrow Re \approx 1.28 \cdot 10^5$

The dimples have a significant impact on Tiger Wood's shot!

Drag coefficient for 2D bodies

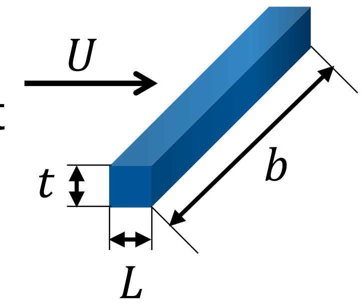
Shape	C_D based on frontal area	Shape	C_D based on frontal area	Shape	C_D based on frontal area
Square cylinder:	2.1	Half cylinder:	1.2	Plate:	2.0
	1.6		1.7	Thin plate normal to a wall:	1.4
Half tube:	1.2	Equilateral triangle:	1.6		
	2.3		2.0	Hexagon:	→ 1.0 ↑ 0.7

Shape	C_D based on frontal area								
Rounded nose section:	L/H:	0.5	1.0	2.0	4.0	6.0			
	C_D:	1.16	0.90	0.70	0.68	0.64			
Flat nose section:	L/H:	0.1	0.4	0.7	1.2	2.0	2.5	3.0	6.0
	C_D:	1.9	2.3	2.7	2.1	1.8	1.4	1.3	0.9

Elliptical cylinder:	Laminar	Turbulent
1:1	1.2	0.3
2:1	0.6	0.2
4:1	0.35	0.15
8:1	0.25	0.1

What does "2D body" mean?

It means that $(t, L) \ll b$ so that "end effects" (will see later) are negligible and the drag coefficient IS NOT a function of b.



This table applies to $Re = \rho U t/\mu \geq 10^4$. In reality, each body has its own $C_D - Re$ characteristic. However, sharp-edged bodies tend to cause separation already at low Re, and once separation occurs, it occurs always at the same place (the sharp edge) and C_D becomes independent of Re.

Note that this table does not account for surface roughness, i.e. bodies are smooth.

Worked example 2

A straight, vertical pole of 10 m height and square cross-section of side 10 cm is immersed in sea water (consider ρ=1000 kg/m^3, μ=0.001 Pa·s) flowing horizontally at 5 m/s. Calculate the drag force acting on the pole if the water flows (a) perpendicular to the face of the pole, or (b) forming a 45 degrees angle with it. Assume that end effects are negligible. (Solution: (a) 26250 N, (b) 28200 N)

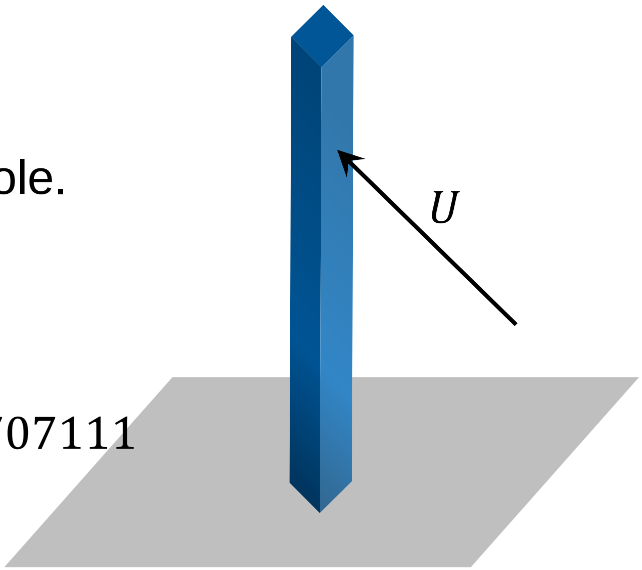
Solution

(a) Side of the pole: $t = 0.1\ m,\qquad Re = \dfrac{\rho U t}{\mu} = 500000$

From the table on the previous slide: $C_D = 2.1$.

Frontal area: $A = tL = 1\ m^2, L = 10\ m$ length of the pole.

Drag force: $D = C_D\left(\frac{1}{2}\rho U^2\right)A = 26250\ N$



(b) Now the characteristic length is $t\sqrt{2}$, $Re = \dfrac{\rho U t\sqrt{2}}{\mu} = 707111$

From the table: $C_D = 1.6$.

The frontal area is now $A = t\sqrt{2}L = 1.41\ m^2 \Longrightarrow D = 28200\ N$

Worked example 3

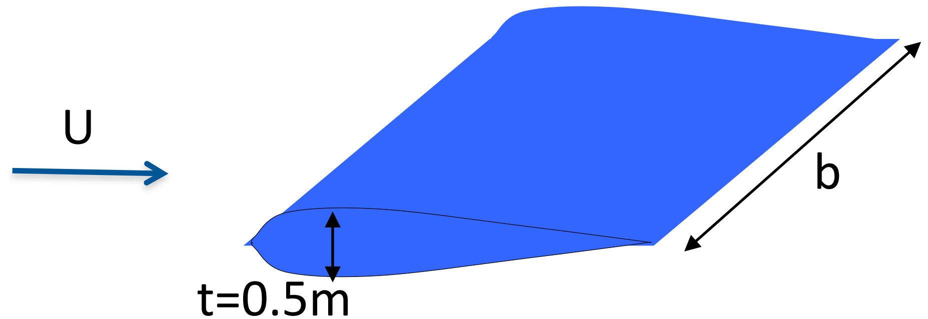
The streamlined airfoil shown below is placed in an airflow and has a drag coefficient of 0.12 based on frontal area when the Reynolds number based on the thickness t is $4 \cdot 10^5$. What diameter d has the circular cylinder of the same length b that has the same drag when placed on the same flow U? Consider that both bodies are very long, $b \gg d$ and $b \gg t$. Air: ρ=1.2 kg/m^3, μ=1.8·10^{-5} Pa·s. (Solution: 0.05 m).

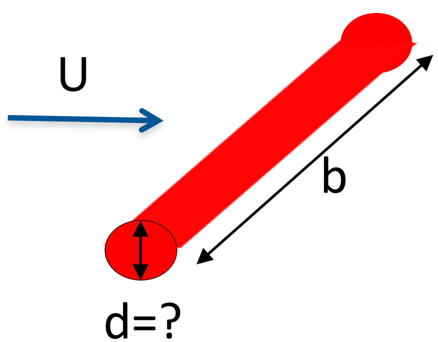
Solution

$$D = C_D\left(\frac{1}{2}\rho U^2\right)bt$$ but we miss U. We can get it from the Reynolds number:

$$Re = \frac{\rho U t}{\mu} \Longrightarrow U = \frac{\mu Re}{\rho t} = 12\ m/s \longrightarrow D = C_D\left(\frac{1}{2}\rho U^2\right)bt = 5.18b$$

We don't know b, but we don't need it





Worked example 3

Now, for the cylinder: $D_{cyl} = C_{D,cyl}\left(\frac{1}{2}\rho U^2\right)bd$

But to extract the diameter we need the drag coefficient. For the drag coefficient we need the Reynolds number for flow past a cylinder, but for this we need the diameter. So, we are in a loop and we need to iterate. Let's start with an initial guess: $d = 0.25\,m$

$$Re_{cyl} = \frac{\rho U d}{\mu} = 200000 \Longrightarrow C_{D,cyl} = 1.2 \quad \text{from the table in slide 21}$$

With this value of C_D, the cylinder diameter that yields the same drag as the airfoil is:

$$D_{airfoil} = C_{D,airfoil}\left(\frac{1}{2}\rho U^2\right)bt = D_{cyl} = C_{D,cyl}\left(\frac{1}{2}\rho U^2\right)bd$$

$$C_{D,airfoil}t = C_{D,cyl}d \Longrightarrow d = \frac{C_{D,airfoil}}{C_{D,cyl}}t = 0.05\,m$$

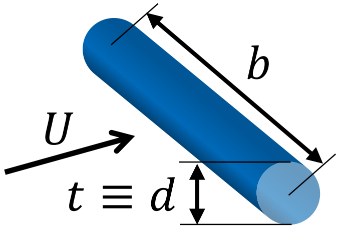
Now that we have an updated value of d, we should recalculate Re_{cyl} and $C_{D,cyl}$:

$$Re_{cyl} = \frac{\rho U d}{\mu} = 40000 \Longrightarrow C_{D,cyl} = 1.2 \quad \text{It did not change}$$

Therefore, the cylinder must be 10 times thinner than the airfoil!!

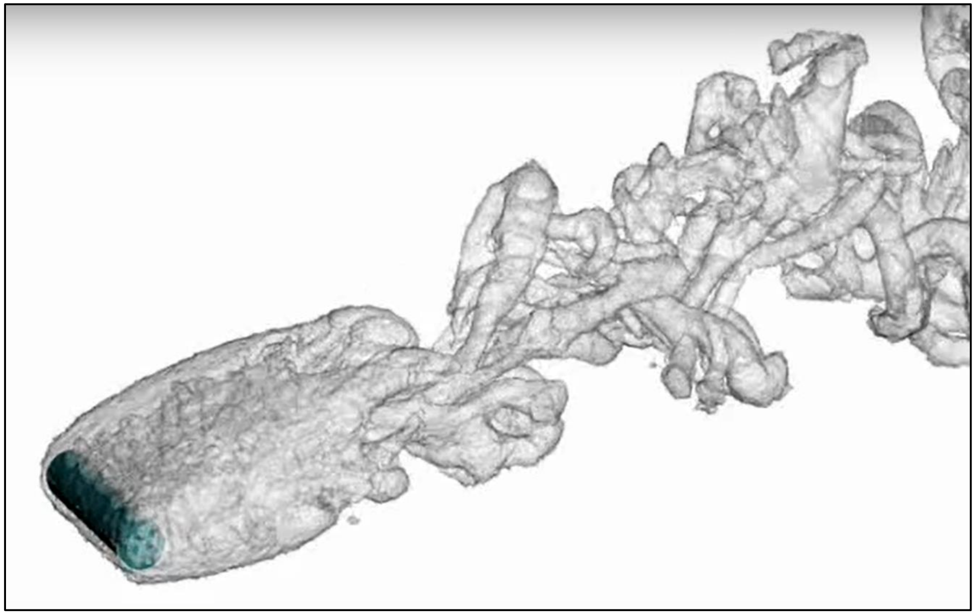
Drag coefficient for 3D bodies

As a “3D body”, we consider bodies whose width b is not sufficiently larger than their thickness or chord length, so that “end effects” occur, making C_D dependent on b.



Example: flow past a finite-length cylinder, $b/d = 5$:

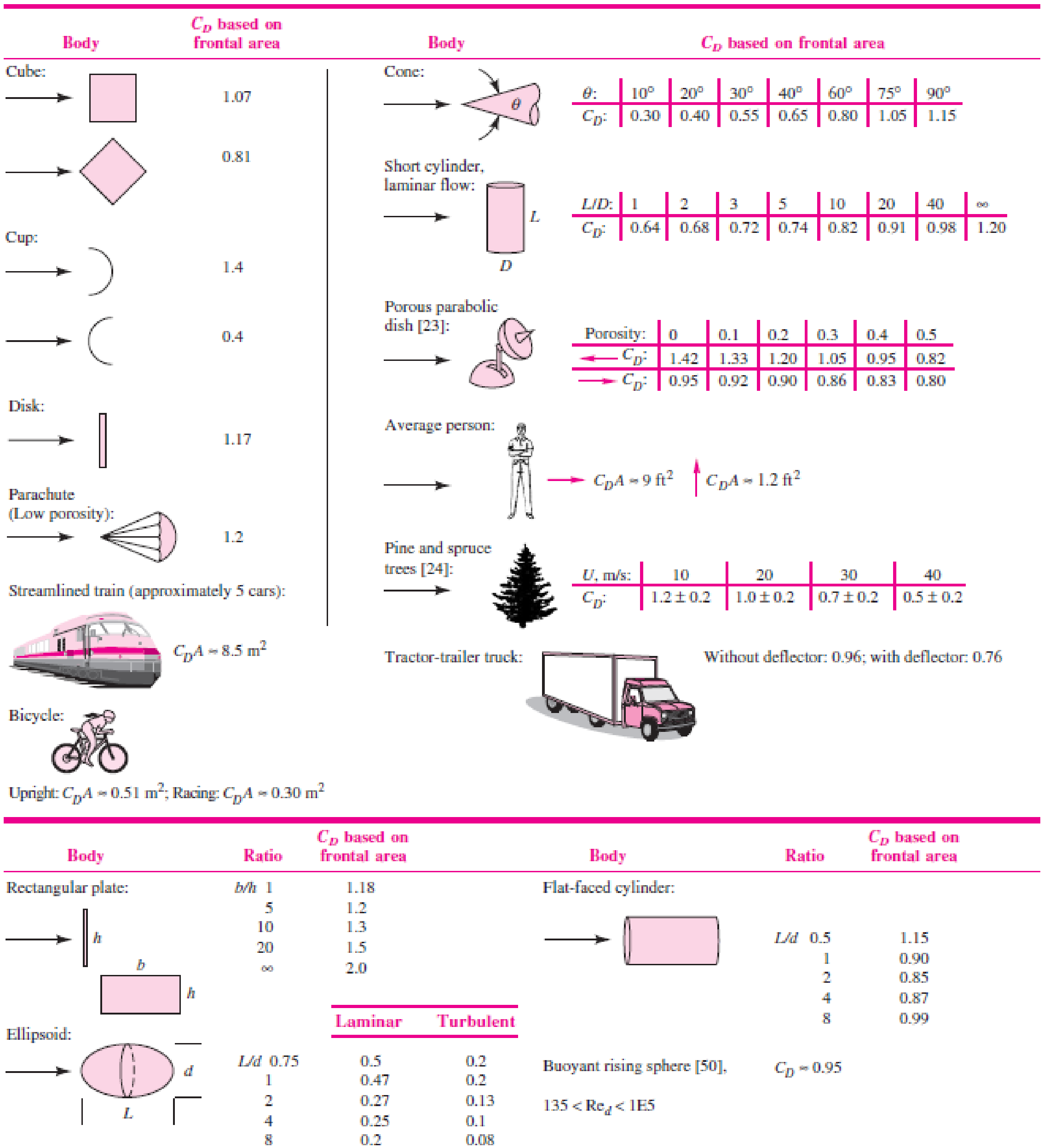
https://www.youtube.com/watch?v=sDMV4yfb7OU&feature=emb_title



End effects reduce pressure drag, so C_D decreases when b/t decreases, example of a finite-length cyilnder:

b/d	1	2	3	5	10	20	40	∞
C_D	0.64	0.68	0.72	0.74	0.82	0.91	0.98	1.2

Drag coefficient for 3D bodies



Body	C_D based on frontal area
Cube:	1.07
Cube (edge-on)	0.81
Cup:	1.4
Cup (reversed)	0.4
Disk:	1.17
Parachute (Low porosity):	1.2
Streamlined train (approximately 5 cars):	$C_DA \approx 8.5\ m^2$
Bicycle:	Upright: $C_DA \approx 0.51\ m^2$; Racing: $C_DA \approx 0.30\ m^2$

Cone:

θ:	10°	20°	30°	40°	60°	75°	90°
C_D:	0.30	0.40	0.55	0.65	0.80	1.05	1.15

Short cylinder, laminar flow:

L/D:	1	2	3	5	10	20	40	∞
C_D:	0.64	0.68	0.72	0.74	0.82	0.91	0.98	1.20

Porous parabolic dish [23]:

Porosity:	0	0.1	0.2	0.3	0.4	0.5
← C_D:	1.42	1.33	1.20	1.05	0.95	0.82
→ C_D:	0.95	0.92	0.90	0.86	0.83	0.80

Average person: $C_DA \approx 9\ ft^2$ (→); $C_DA \approx 1.2\ ft^2$ (↑)

Pine and spruce trees [24]:

U, m/s:	10	20	30	40
C_D:	1.2 ± 0.2	1.0 ± 0.2	0.7 ± 0.2	0.5 ± 0.2

Tractor-trailer truck: Without deflector: 0.96; with deflector: 0.76

Body	Ratio	C_D based on frontal area
Rectangular plate:	b/h 1	1.18
	5	1.2
	10	1.3
	20	1.5
	∞	2.0

Body	Ratio	Laminar	Turbulent
Ellipsoid:	L/d 0.75	0.5	0.2
	1	0.47	0.2
	2	0.27	0.13
	4	0.25	0.1
	8	0.2	0.08

Body	Ratio	C_D based on frontal area
Flat-faced cylinder:	L/d 0.5	1.15
	1	0.90
	2	0.85
	4	0.87
	8	0.99
Buoyant rising sphere [50], $135 < Re_d < 1E5$		$C_D \approx 0.95$

This table applies to $Re = \rho Ut/\mu \geq 10^4$. As for the table in slide 21, in reality each body has its own $C_D - Re$ characteristic. However, sharp-edged bodies tend to cause separation already at low Re, and once separation occurs, it occurs always at the same place (the sharp edge) and C_D becomes independent of Re.

Note that this table does not account for surface roughness, i.e. bodies are smooth.

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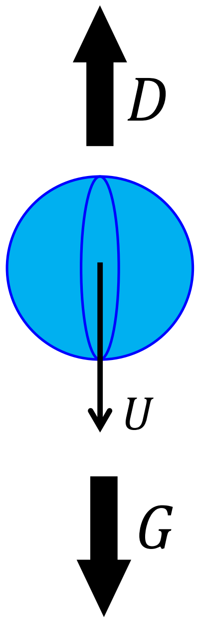
Worked example 4

Calculate the terminal speed (when gravity balances drag) of a rain droplet of $d = 4\ mm$, by assuming that the droplet has a perfect spherical shape.



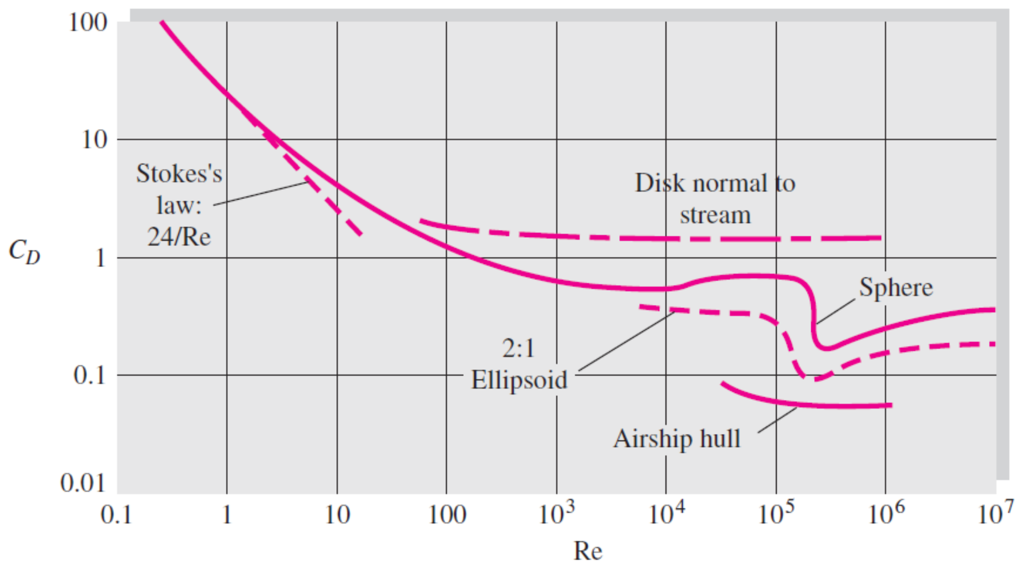
Solution

$$drag: D = \frac{1}{2} C_D \rho_{air} U^2 \frac{\pi d^2}{4}$$

$$gravity: G = g(\rho_{water} - \rho_{air}) \frac{\pi d^3}{6}$$

$$D = G \Longrightarrow \frac{1}{2} C_D \rho_{air} U^2 \frac{\pi d^2}{4} = g(\rho_{water} - \rho_{air}) \frac{\pi d^3}{6}$$

$$U = \sqrt{\frac{4}{3} \frac{gd}{C_D} \frac{\rho_w - \rho_a}{\rho_a}}$$



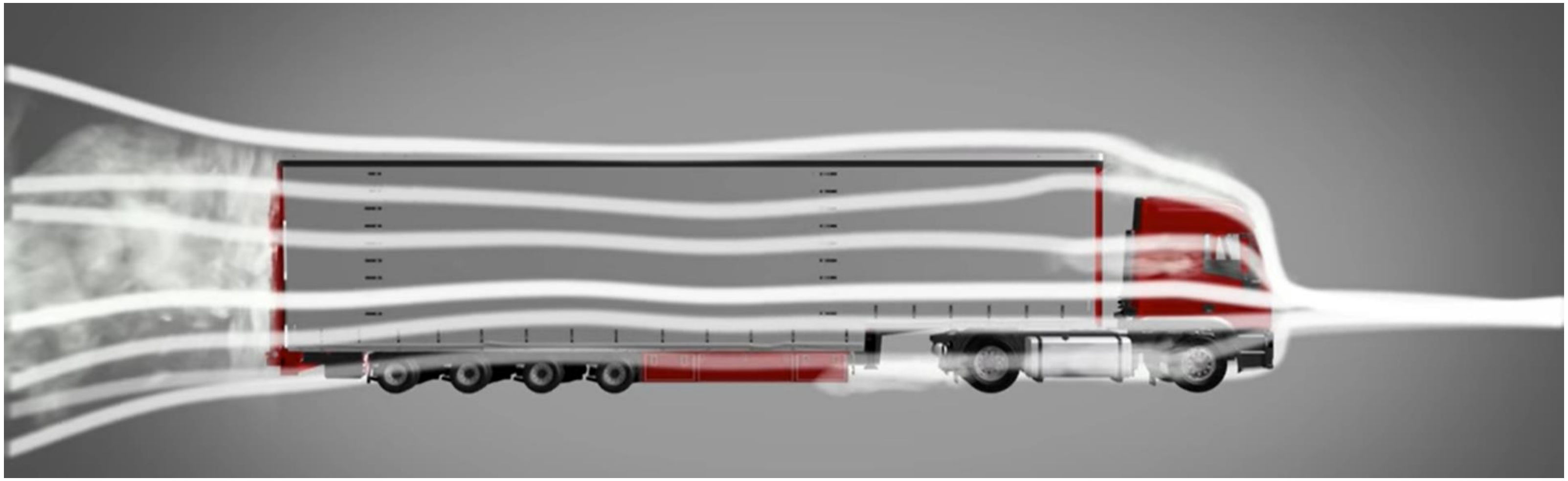
For C_D, we need Re, therefore U. Let's start with a guess value, $Re = 10^4$, i.e. $C_D = 0.47$ (slide 26), $\Longrightarrow U = 9.6\ m/s$. This yields $Re = \rho_a U d/\mu_a = 2560$. C_D is essentially the same, so we can conclude that $U = 9.6\ m/s$.

Drag reduction in road vehicles

Drag reduction technology is essential in the design of road vehicles, to save fuel and improve vehicle performance:

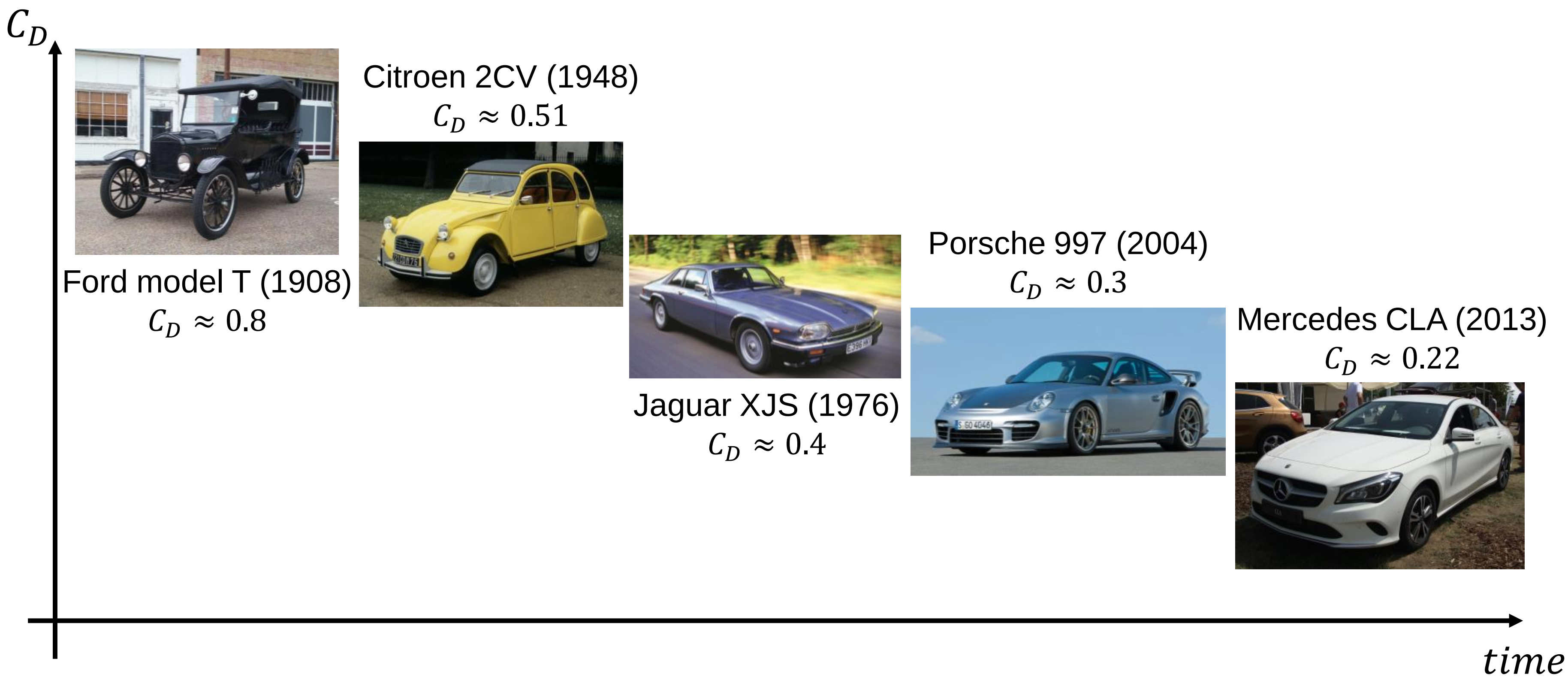
Example: drag over a truck

https://www.youtube.com/watch?time_continue=125&v=jOG6RSjIEEs&feature=emb_logo



Drag reduction in road vehicles

Evolution of cars aerodynamics (https://en.wikipedia.org/wiki/Automobile_drag_coefficient)



Interestingly, F1 cars have

$C_D = 0.75\ (Monza\ GP) - 1.25\ (Monaco\ GP)$

Summary of drag reduction strategies

$$C_D = C_{D,friction} + C_{D,pressure}$$

Reduction of friction drag

- Minimise planform area
- Maintain a laminar boundary layer
- Minimise surface roughness

Reduction of pressure drag

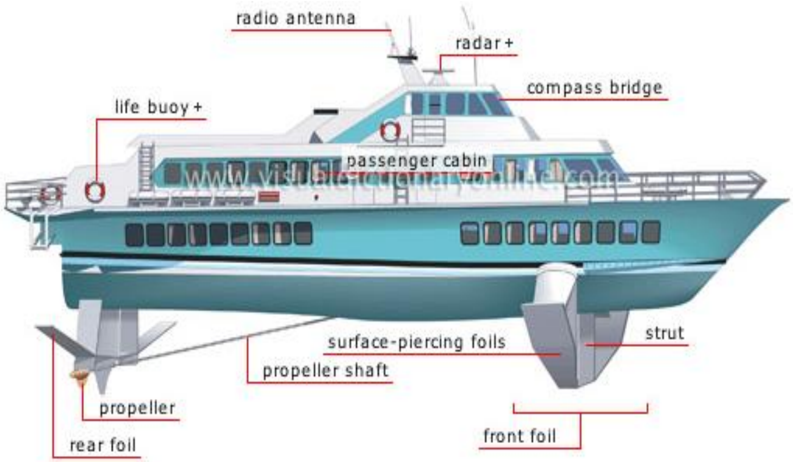
- Streamline the body (which increases planform area)
- Minimise frontal area
- Retard flow separation
- Try to induce turbulence
- Make surface rough (dimples)

In summary, the strategies decreasing one form of drag tend to increase the other. The best solution is a compromise between the two, to be found depending on which drag form is more important (according to body shape and Reynolds number).

Lift force

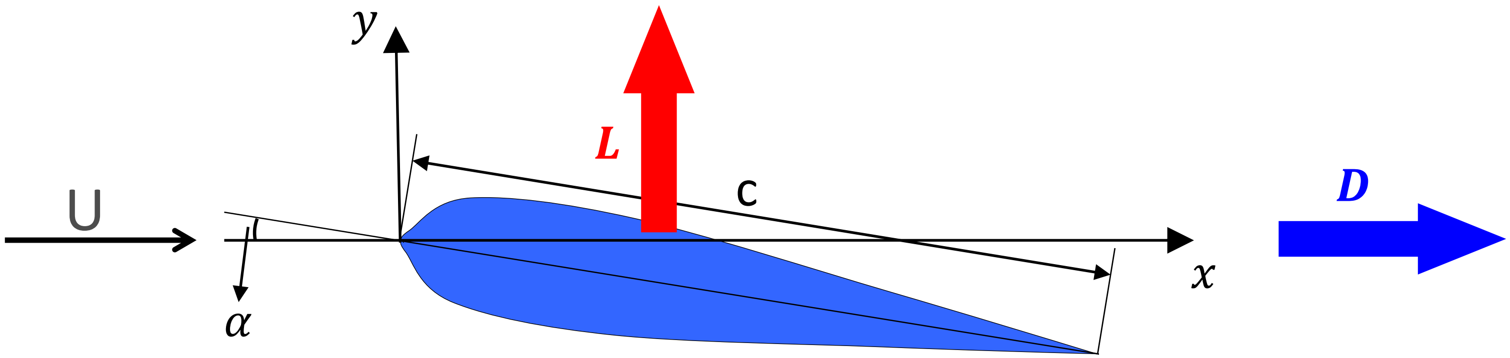
The lift force is <u>orthogonal</u> to the flow direction.

The lift force is generated by lift bodies such as airfoils, hydrofoils, wings, carefully-designed to <u>maximise lift</u> and <u>minimise drag</u>.

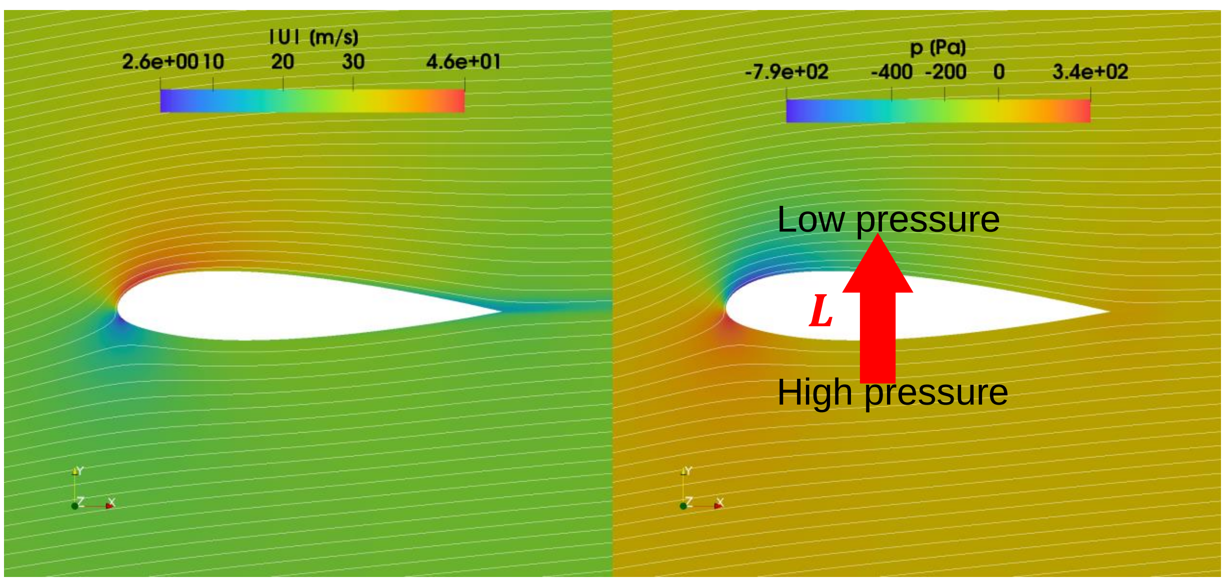


Lift force

For a symmetric airfoil, in order to generate lift, the chord line needs to form a non-zero angle α with the free-stream velocity. α is the **angle of attack**.

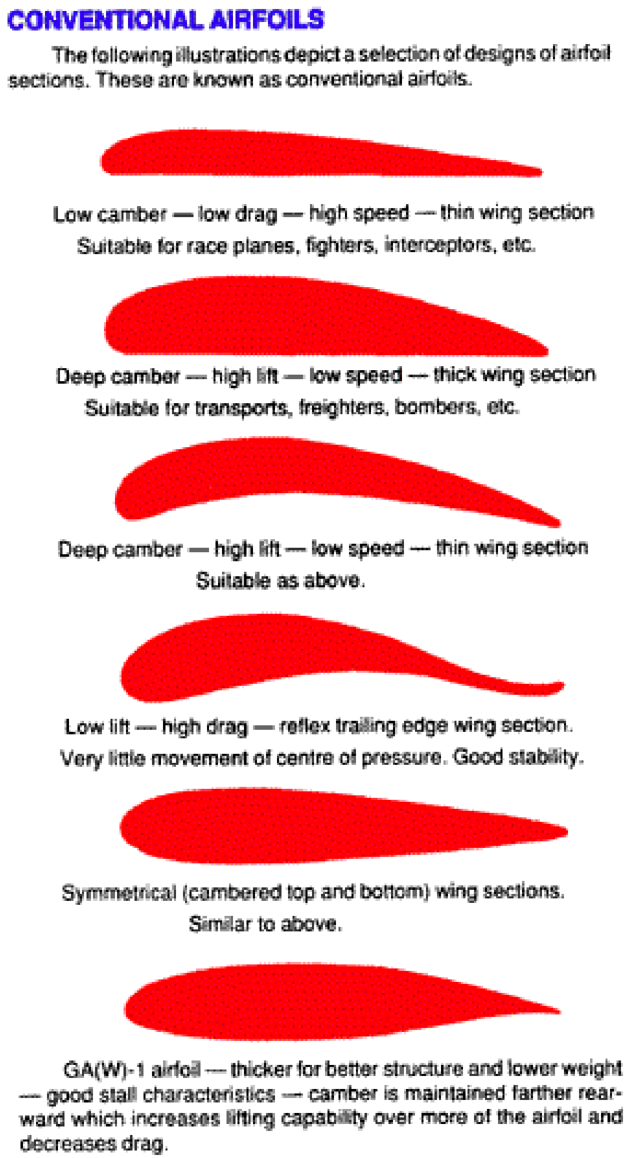


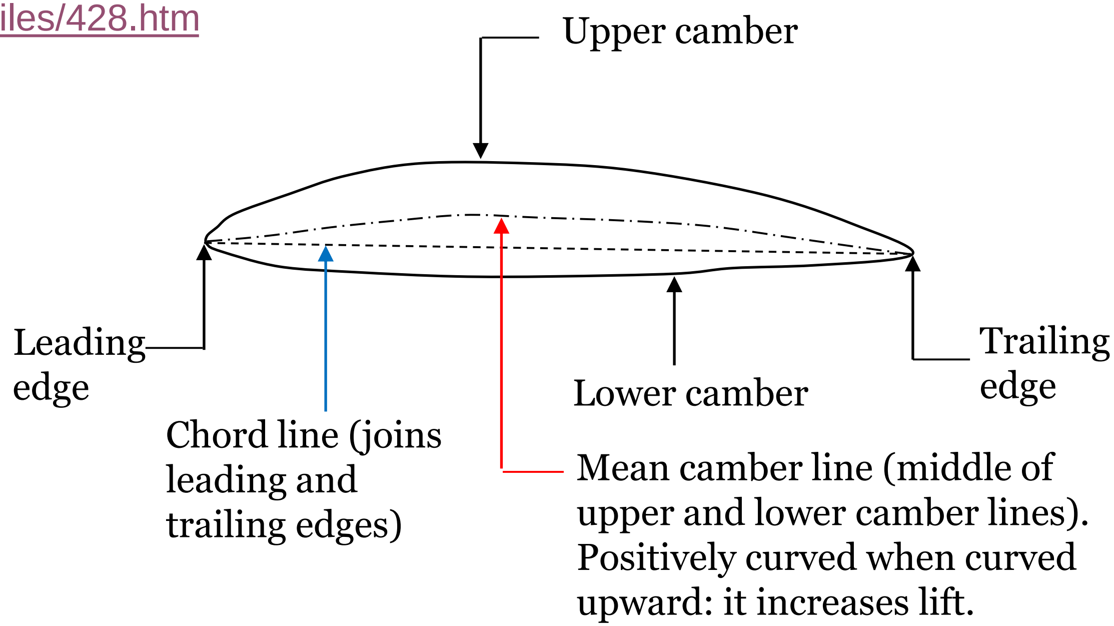
Why is lift generated? Because of the pressure difference between upper and lower surfaces



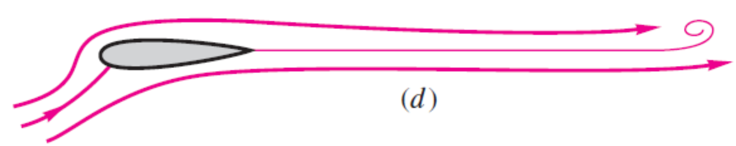
Airfoil definitions

From: http://mnabil77.tripod.com/projectfiles/428.htm

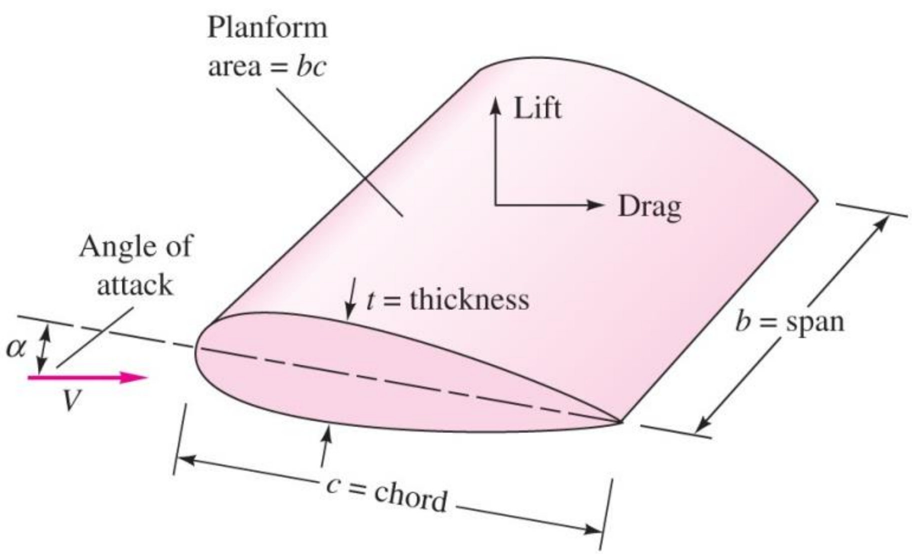




- The thickness to chord ratio is usually $t/c \leq 0.25$
- A cambered airfoil generates lift even when $\alpha = 0$
- Rounded leading edge to prevent flow separation
- Sharp trailing edge to cause a tangential wake motion that generates lift



Lift coefficient



For airfoils, the characteristic area is the planform area $A_p = bc$

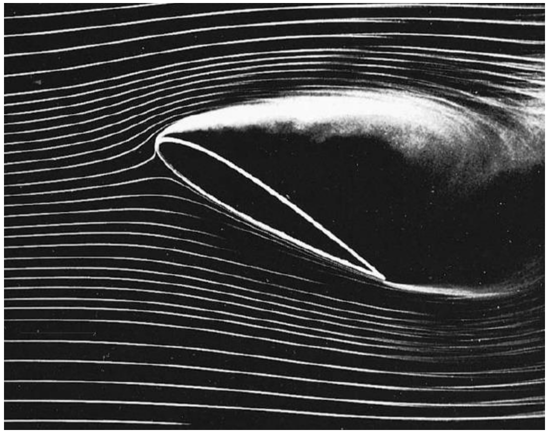
Drag coefficient $$C_D = \frac{D}{\frac{1}{2}\rho U^2 A_p}$$

Lift coefficient $$C_L = \frac{L}{\frac{1}{2}\rho U^2 A_p}$$

In principle, $C_D, C_L = C_D, C_L(\alpha, Re_c)$, but Re_c is usually in the turbulent boundary layer range and has a modest effect.

Remember that we want to generate lift. In general, C_L (but also C_D) increases with the angle of attack up to a max α, usually $\alpha = 15 - 20^o$, when flow separates from the upper surface and the airfoil stalls. Further increase of α leads to great drop of C_L and increase of C_D.

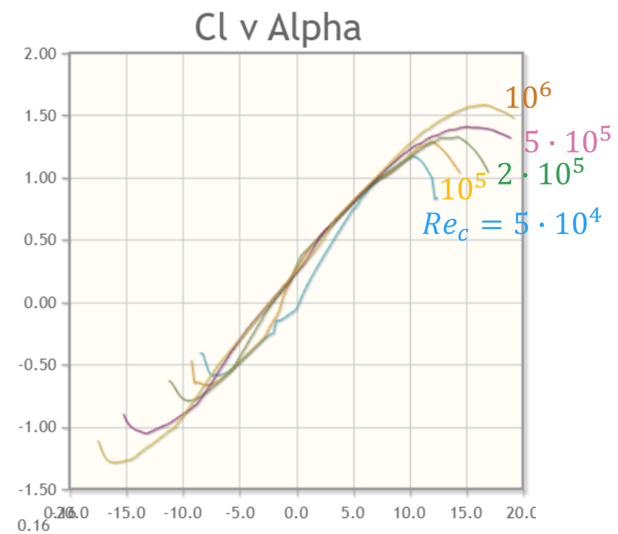
A stalled airfoil



Lift and drag charts for airfoils

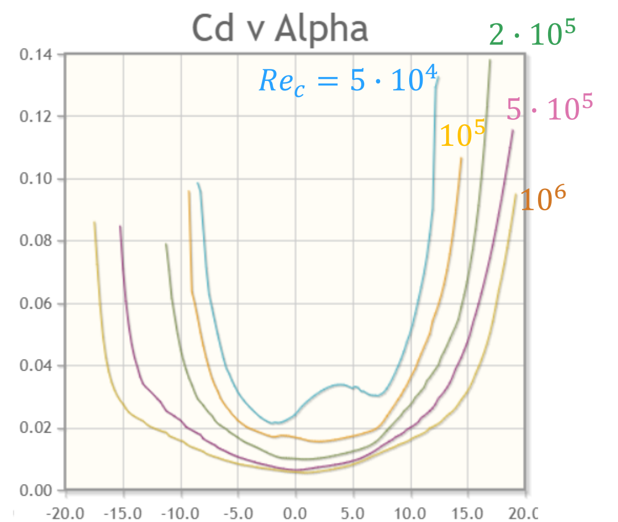
Lift and drag charts can be found in textbooks or online, check out this webpage:

http://airfoiltools.com/airfoil/details?airfoil=naca2412-il

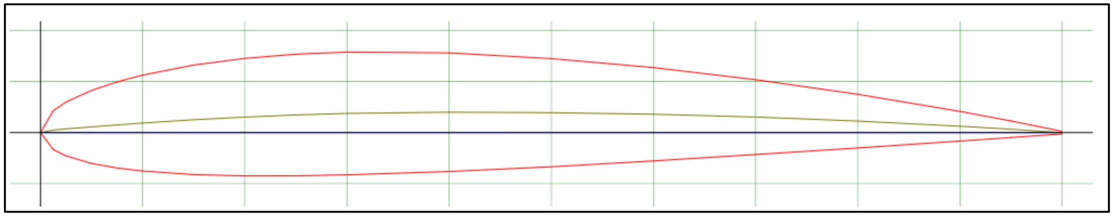


Many airfoils have been designed and tested by NACA (forerunner of NASA), and follow a 4-digits (up to 8-digits) nomenclature: NACA xyzz, where:

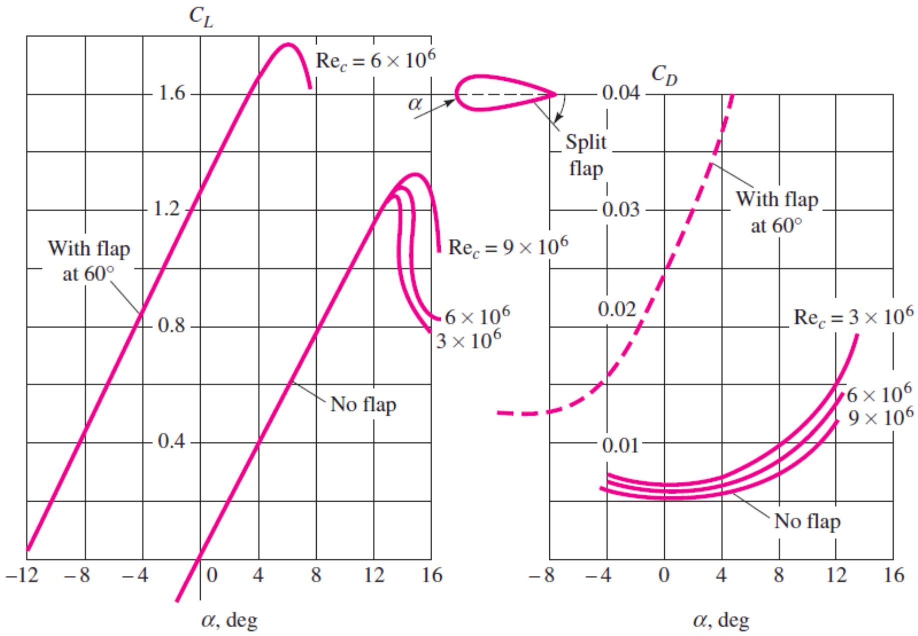
- x: max camber as percentage of the chord.
- y: distance of max camber point from leading edge in tens of percent of the chord length.
- zz: maximum thickness of airfoil as percent of chord.



NACA 2412



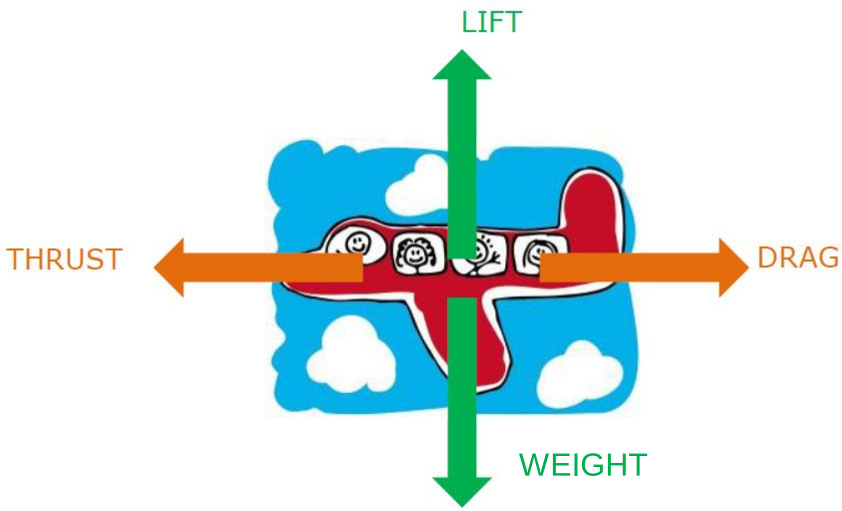
Use of flaps



Flaps induce positive variations of the airfoil camber, thus increasing C_L. Very useful for take-off or landing, when the speed of the airplane (thus Re_c) is too small for the airfoil to generate sufficient lift force to lift the airplane (take-off) or let it descend gently (landing).

Steady horizontal flight

For an aircraft (or any flying object) in steady horizontal flight, there are no accelerations and all the forces acting on the aircraft balance.



- The weight must be balanced by the lift:

$$W = L = C_L \left(\frac{1}{2}\rho U^2\right) A_p$$

- The thrust must overcome the drag:

$$T = D = C_D \left(\frac{1}{2}\rho U^2\right) A_p$$

With the simplifying assumption that all the lift and drag contributions come from the wings only, the area to be used in the equations above is the total planform area of the wings.

Worked example 5

An Embraer E190 has a mass of 47,000 kg.

(a) What is the lift force when the jet cruises in steady horizontal flight at 500 km/h at an altitude of 5000 m?

(b) Assuming all the lift comes from the wings calculate the lift coefficient given that the wing area is 92.5 m^2.

(c) If the drag coefficient is 3% of the lift coefficient, what is the thrust required?

(d) What thrust power are the engines generating at this flight condition?

Embraer E190
http://www.flugzeuginfo.net/acimages/emb190_wang_t.jpg

Worked example 5

Solution

(a) If the jet cruises in steady horizontal flight, the lift force must be equal to the weight:

$$L = W = mg = 461070\ N$$

(b) Lift coefficient: $C_L = \dfrac{L}{\frac{1}{2}\rho U^2 A_p}$ $\qquad U = 500\ km/h = 138.9\ m/s$

We need the air density at 5000 m. International Standard Atmosphere (ISA) table:

https://onlinelibrary.wiley.com/doi/pdf/10.1002/9781118534786.app1

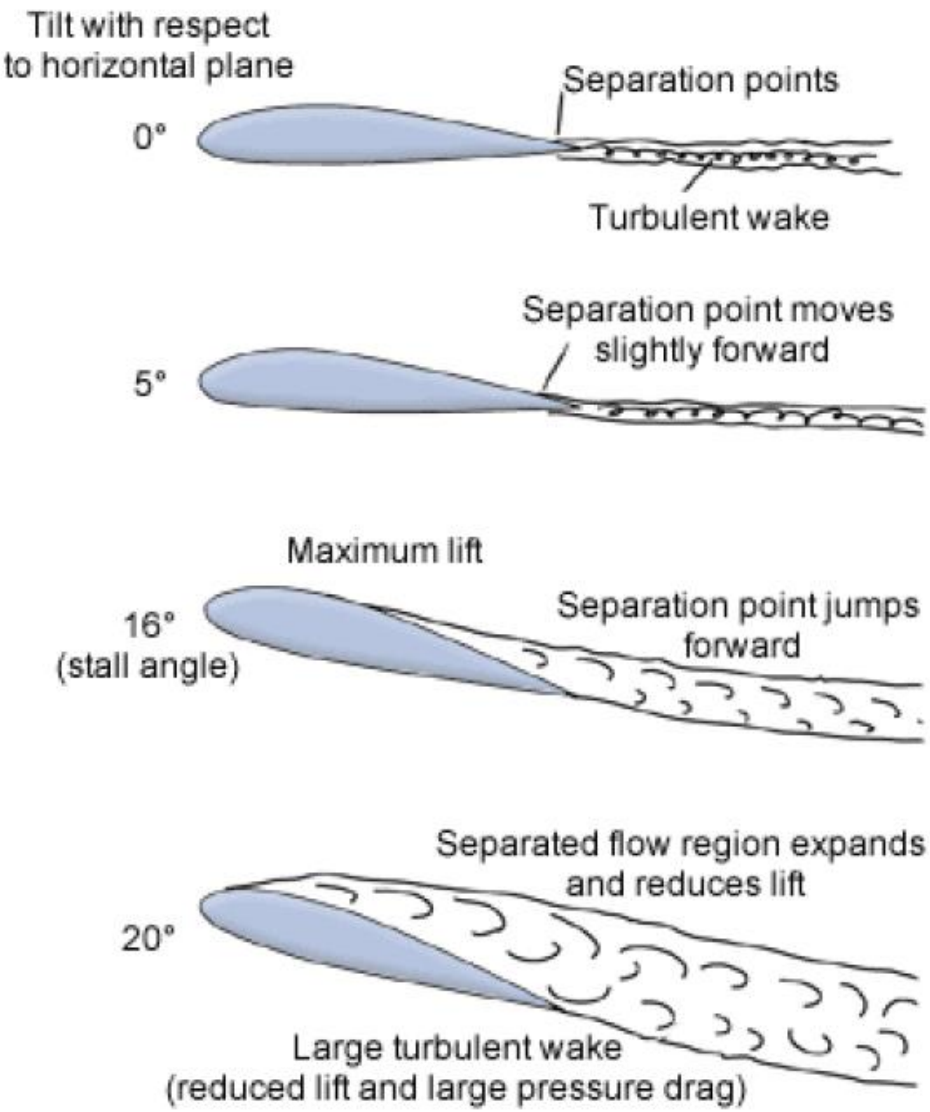
$\rho(5000\ m) \approx 0.736\ \ kg/m^3$

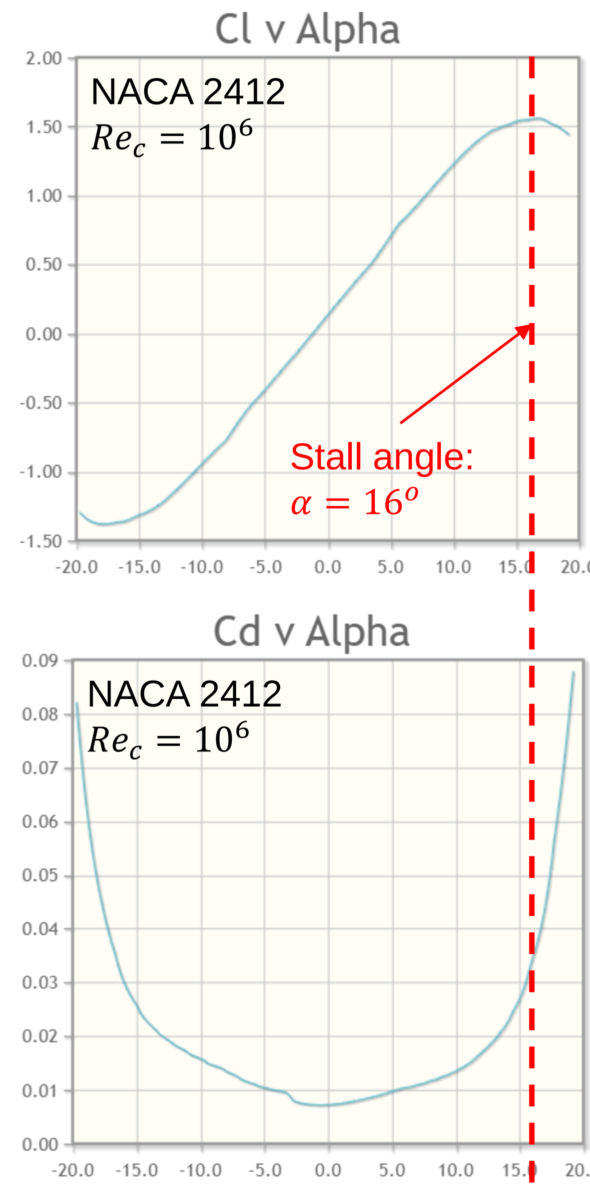
→ $C_L = 0.702$

c) The drag coefficient is said to be 3% of the lift, therefore $D = 0.03L = 13832\ N$. In steady flight, the thrust balances drag and therefore $T = 13832\ N$

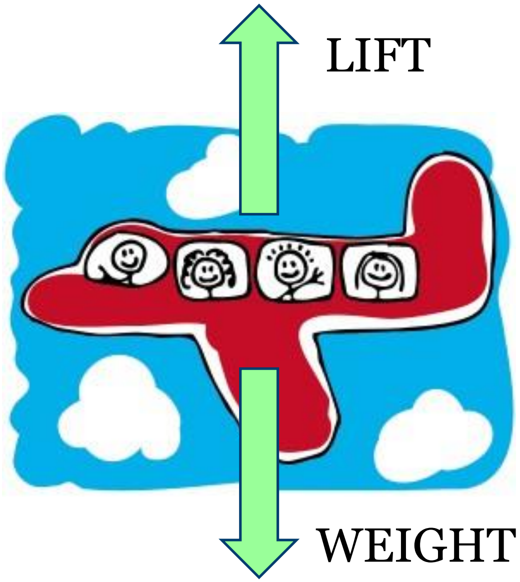
d) Thrust power: $P = T \cdot U = 1921\ kW$

Stall


Tilt with respect
to horizontal plane
Separation points
0°
Turbulent wake
Separation point moves
slightly forward
5°
Maximum lift
16°
(stall angle)
Separation point jumps
forward
Separated flow region expands
and reduces lift
20°
Large turbulent wake
(reduced lift and large pressure drag)


Cl v Alpha
NACA 2412
$Re_c = 10^6$
Stall angle:
$\alpha = 16^o$
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-20.0
-15.0
-10.0
-5.0
0.0
5.0
10.0
15.0
Cd v Alpha
NACA 2412
$Re_c = 10^6$
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00

Stall speed



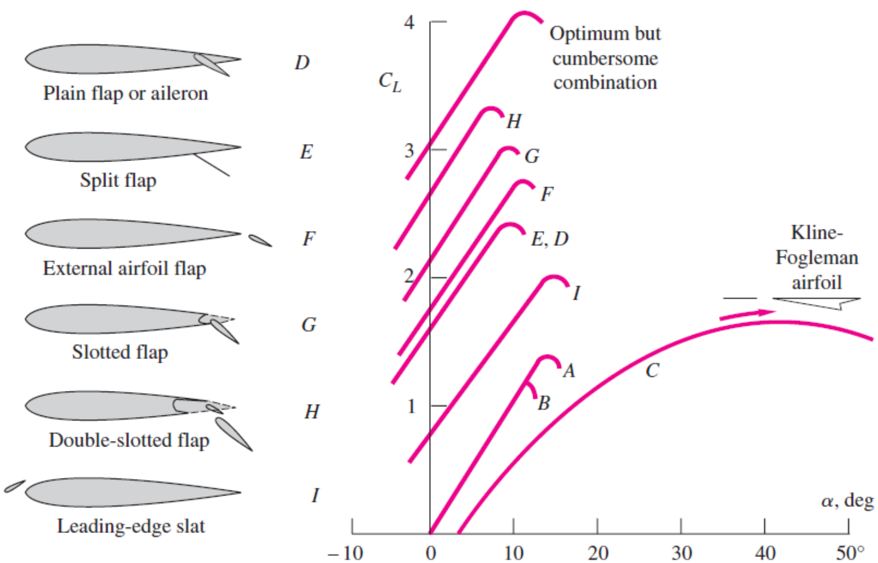
For steady horizontal flight conditions, the lift generated by the airfoil must balance the weight of the aircraft:

$$W = L = C_L \left(\frac{1}{2}\rho U^2\right) A_p$$

The existence of a $C_{L,max}$ for an airfoil implies the existence of a minimum speed, stall speed U_{stall}, for the airfoil to generate sufficient lift force to sustain the aircraft weight:

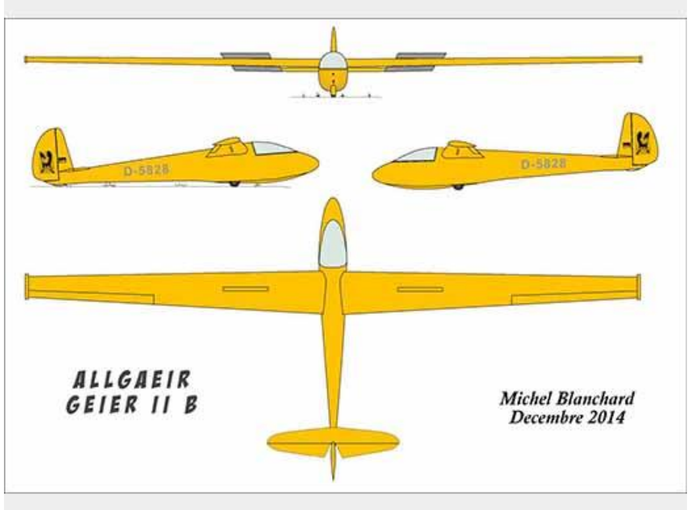
$$U_{stall} = \left(\frac{2W}{C_{L,max}\rho A_p}\right)^{1/2}$$

For safety reasons, the speed of an aircraft must always be above $1.2U_{stall}$ to avoid instability and stall. To secure sufficient lift at slow speed, for example when landing, aircrafts use flaps (see figure).



Worked example 6

The Allgeier Geier 2 glider (https://www.j2mcl-planeurs.net/dbj2mcl/planeurs-machines/planeur-fiche_0int.php?code=625) employs NACA 63-618 airfoils, has a wing span of 17.76 m, a total wing area of 14 m^2 and a weight of 370 kg. Calculate its minimum safe landing speed.



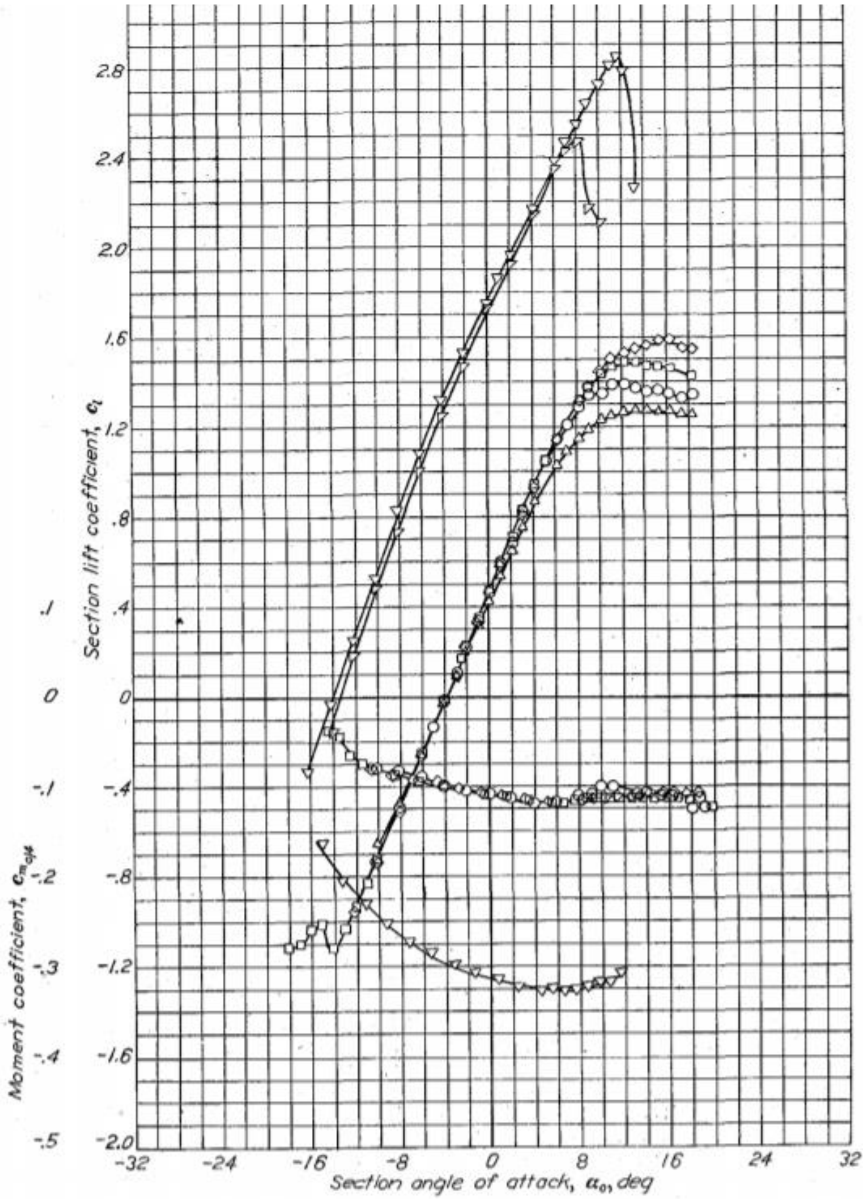
Sources of lift/drag coefficients data for NACA:

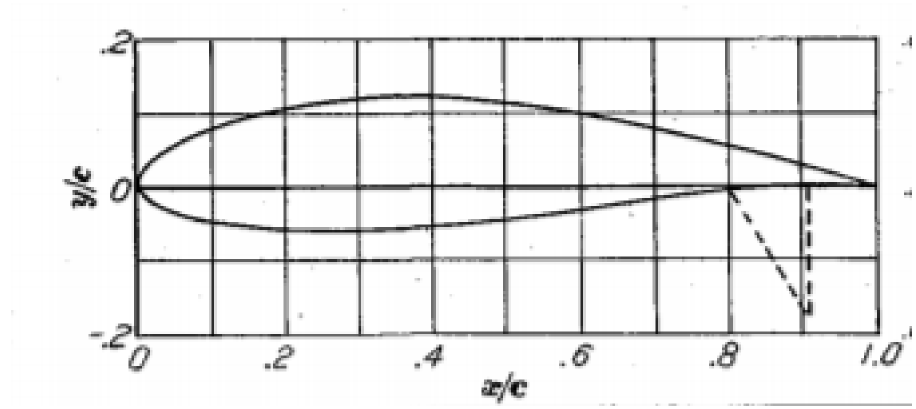
http://airfoiltools.com/airfoil/details?airfoil=naca633618-il

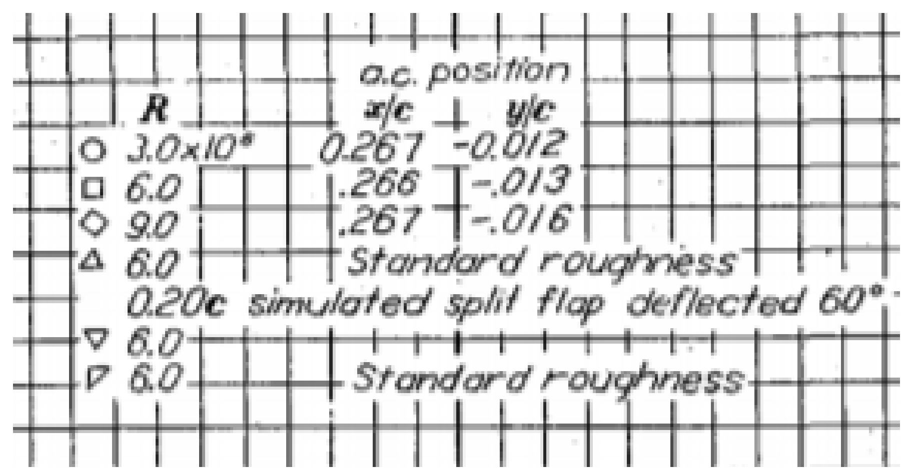
https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930090976.pdf (page 173)

Worked example 6

https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930090976.pdf (page 173)







Worked example 6

Solution

We need the stall speed: $U_{stall} = \left(\frac{2W}{C_{L,max}\rho A_p}\right)^{1/2}$

From the chart on the previous slide, which curve for C_L shall we consider? We need to know the Reynolds number (and if the surface is smooth/rough), which means speed (unknown) and chord length. Let's start with a guess value of $Re_c = 3 \cdot 10^6$, available in the chart, and assume a smooth airfoil. From the chart (circles), we get $C_{L,max} \approx 1.4$, therefore (take air density at terrestrial level: 1.2 kg/m³):

$$U_{stall} = \left(\frac{2 \cdot 370\,kg \cdot 9.81\,\frac{m}{s^2}}{1.4 \cdot 1.2\,\frac{kg}{m^3} \cdot 14\,m^2}\right)^{1/2} = 17.6\,m/s \Longrightarrow U = 1.2U_{stall} = 21.1\,m/s$$

We can now check the Reynolds number. We need the chord length, we can estimate this as the wing area divided by the span, $c = 14\,m^2/17.76\,m = 0.79\,m$. This gives $Re_c = \rho U c/\mu \approx 10^6$. We should now recalculate $C_{L,max}$. We don't have it on the chart, however, we can assume that it won't be too different from that calculated with $Re_c = 3 \cdot 10^6$, so we can conclude that the minimum safe landing speed is $21.1\,m/s$.

Finite span wings – wingtip vortices

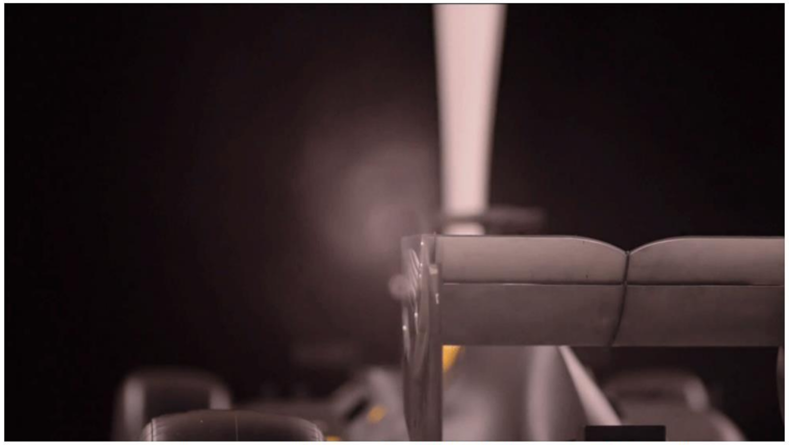
All the data (C_D, C_L) for airfoils are presented for infinite length (2D) airfoils. In reality, airfoils have finite span and this causes wingtip trailing vortices in the aircraft wake, that are often visible.

https://www.youtube.com/watch?v=dfY5ZQDzC5s

https://www.f1technical.net/features/21854

Variations of vortices: vicious or virtuous?

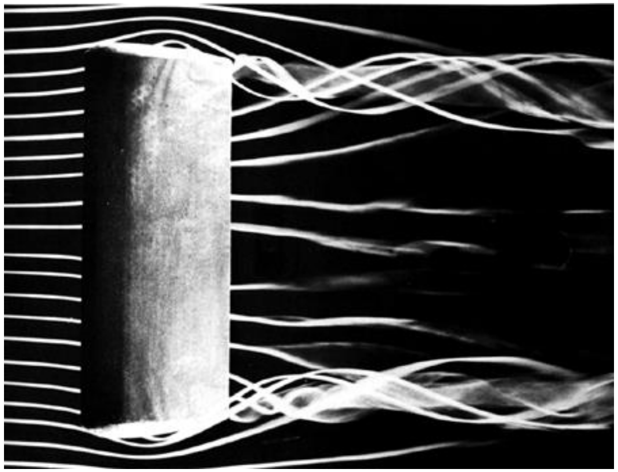
By Vyssion and jjn9128 on 24 Sep 2018, 10:00

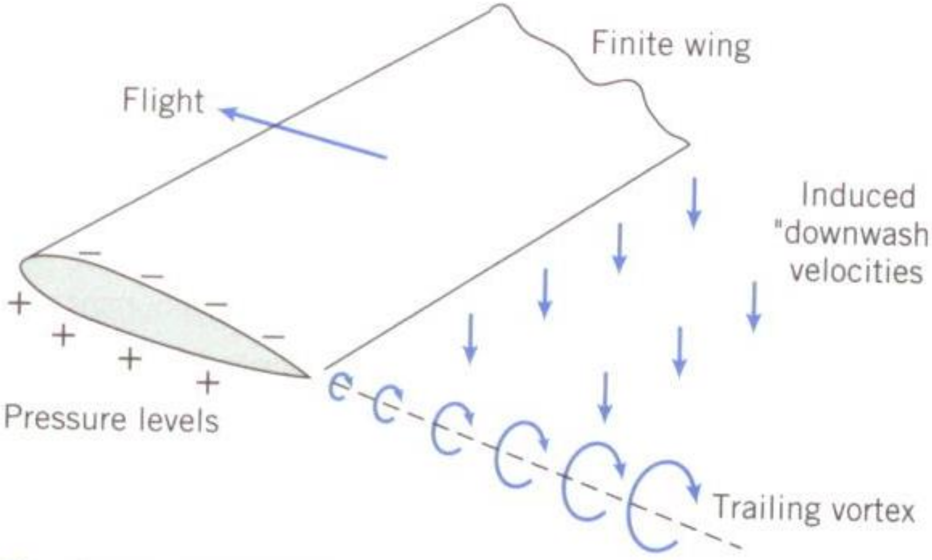


Smoke visualizing rear wing tip vortex in wind tunnel, source: http://techf1les.files.wordpress.com/20 ... d-drag.gif

Finite span wings – wingtip vortices

The vortices occur because, at the wingtip, the pressure on the lower surface is larger than that on the upper surface, so that air curls upward around the wingtip. With the movement of the airplane, these vortices stream out behind the wing creating trailing vortices and inducing downwash velocities. Overall, the vortices **increase drag and reduce** lift on the airfoil!





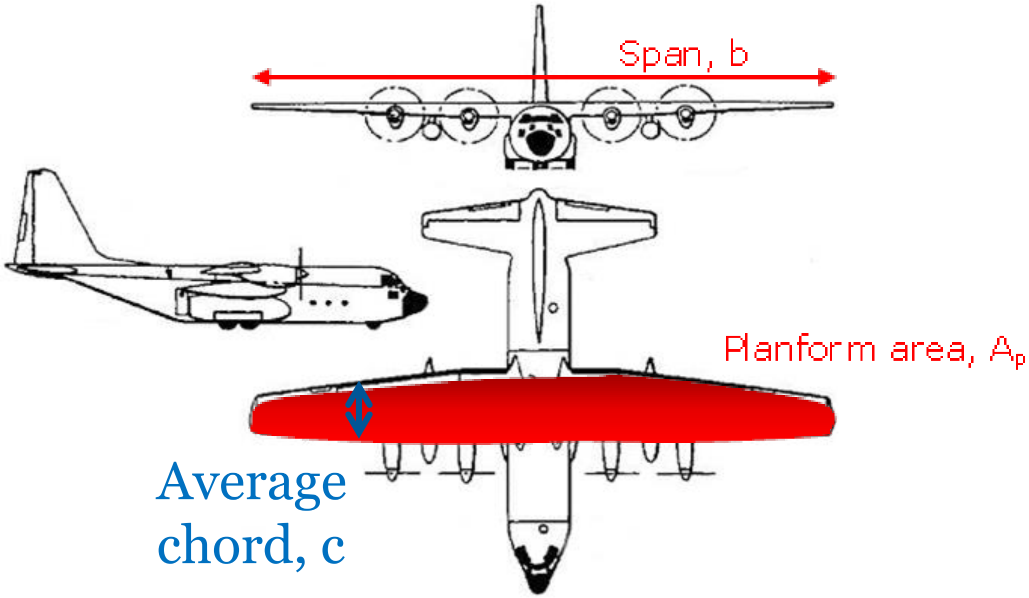
Finite span wings – wingtip vortices

The effect of finite span is correlated with the wing aspect ratio:

$$AR = \frac{b^2}{A_p} = \frac{b^2}{bc} = \frac{b}{c}$$

Compared to the infinite airfoil, the angle of attack for a finite-length airfoil has to increase by $\Delta\alpha$ to get the same lift, with:

$$\Delta\alpha = \frac{C_L}{\pi AR}$$



The associated drag increase is:

$$\Delta C_D \approx C_L \Delta\alpha = \frac{C_L^2}{\pi AR}$$

so that:

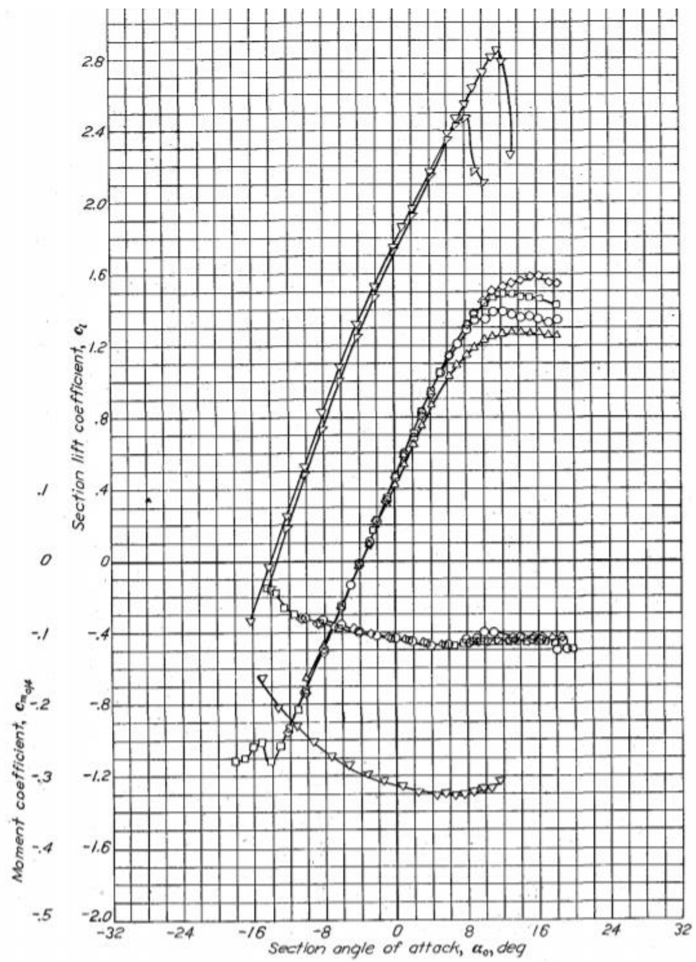
$$C_D = C_{D,\infty} + \frac{C_L^2}{\pi AR}$$

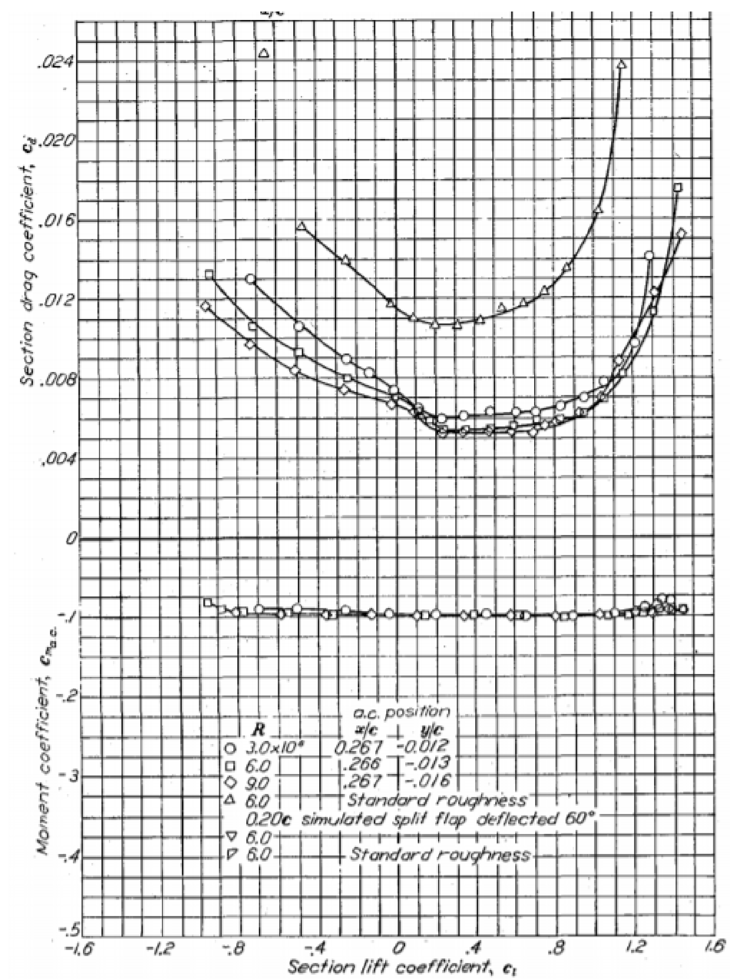
Finite span wings – wingtip vortices

Methods to weaken trailing vortices: **endplates** and **winglets**

Worked example 7

Consider the Allgeier Geier 2 glider of Worked example 6. By how much must the angle of attack change from the infinite case for a C_L of 1.1? (use the triangle symbols). What is the drag coefficient of this wing at this condition?





Worked example 7

Solution

The angle of attack must change by: $\Delta\alpha = \dfrac{C_L}{\pi AR}$

The aspect ratio of the wing is: $AR = \dfrac{b^2}{A_p} = \dfrac{(17.76\ m)^2}{14\ m^2} = 22.53$

Therefore: $\Delta\alpha = \dfrac{1.1}{\pi\ 22.53} = 0.0155\ rad = 0.9\ deg$

Resulting drag coefficient: $C_D = C_{D,\infty} + \dfrac{C_L^2}{\pi AR}$

From the C_D chart (triangles), we see that $C_{D,\infty} = 0.019$ when $C_L = 1.1$. Therefore:

$$C_D = 0.019 + \frac{1.1^2}{\pi\ 22.53} = 0.036$$

Worked example 8

Exam paper 2016/17 – Fluids, long question

On a Formula One car a wing is mounted on the back just at the exhaust pipe exit. The exhaust gases exit the exhaust pipe with a velocity of 100 m/s and a temperature of 819 K.

Air density at 0 degree Celsius is 1.2 kg/m^3. For this calculation assume that the exhaust gases have the same properties of air with R = 287 J/(kg·K) and specific heat at constant pressure c_p = 1004 J/kg·K.

(a) Considering that the wing has a planform area of 0.01 m^2 with a chord of 0.02 m and the lift force coefficient is 1.25, estimate the lift force generated. [3]

(b) Calculate the total drag coefficient of the finite span wing considering that the drag force coefficient for the 2D profile (infinite span) is 0.2. [3]

There was also c) and d) but these have not been covered yet…

Worked example 8

Solution

(a) $$L = C_L\left(\frac{1}{2}\rho U^2\right)A_p$$

We need to calculate the density of the gas at 819 K. To this aim, we can use the ideal gas law:

$$p\frac{1}{\rho} = RT \Longrightarrow \rho T = \frac{p}{R} = constant$$

$$\rho(T = 273\ K)\cdot 273\ K = \rho(T = 819\ K)\cdot 819\ K$$
$$\Longrightarrow \rho(T = 819\ K) = 1.2\,\frac{kg}{m^3}\cdot\frac{273\ K}{819\ K} = 0.4\ \frac{kg}{m^3}$$

$$\Longrightarrow L = 1.25\left(\frac{1}{2}\cdot 0.4\ \frac{kg}{m^3}\cdot\left(100\frac{m}{s}\right)^2\right)0.01\ m^2\ = 25\ N$$

(b) $$C_D = C_{D,\infty} + \frac{C_L^2}{\pi AR}, \qquad AR = \frac{b}{c} \qquad b = \frac{A_p}{c} = \frac{0.01\ m^2}{0.02\ m} = 0.5\ m \Longrightarrow AR = 25$$

$$\Longrightarrow C_D = 0.2 + \frac{1.25^2}{\pi 25} = 0.22$$

Topic 3 – Lift and drag

Now you should be able to:

- Recognise the difference between friction and pressure drag
- Relate drag coefficient and drag force via correct identification of the representative area, and of the length scale used to calculate the Reynolds number
- Use the $C_D - Re$ chart for flow past cylinder/sphere, or tables for more complex 2D and 3D bodies, to extract drag coefficients and perform calculations
- Identify strategies to reduce drag on bodies immersed in fluid flow
- Relate lift coefficient and lift force for airfoils
- Extract lift/drag coefficients for airfoils from charts, depending on the angle of attack
- Apply the force balance for a body flying horizontally at steady conditions
- Evaluate the minimum speed to avoid airfoil stall
- Discuss the impact of wingtip vortices on finite-span wings

Further reading/assessment:

- F. White book, Sec. 7.6 and examples therein; problems in Ch. 7; notes/exercises in Moodle

Seminar

Worked example 9

Exam paper 2017/18 – Fluids, long question

A kite flies in the air with a string held by a child on the ground. The mass of the kite is 0.05 kg and the length of its string is 1.0 m and width is 0.5 m. The air flows horizontally at a velocity of 15 km/h.

The air density and viscosity may be taken as:
$\rho = 1.2\ kg/m^3$, $\mu = 1.8 \times 10^{-5}\ kg/ms$

(a) If the kite could be assumed as a flat plate, explain why it can't be in equilibrium in the air when its orientation is parallel to the horizontal. [1]

(b) Consider the situation in which the kite flies with the angle of the string to the horizontal direction at 45° and the tension in the string at 50 N. Sketch the directions of all force on the kite under this situation on your answer booklet. [1]

(c) With the aid of the sketch from Q13 (b), calculate the drag and lift forces on the kite when it is in equilibrium perpendicular and parallel to the ground with the string at 45°. [4]

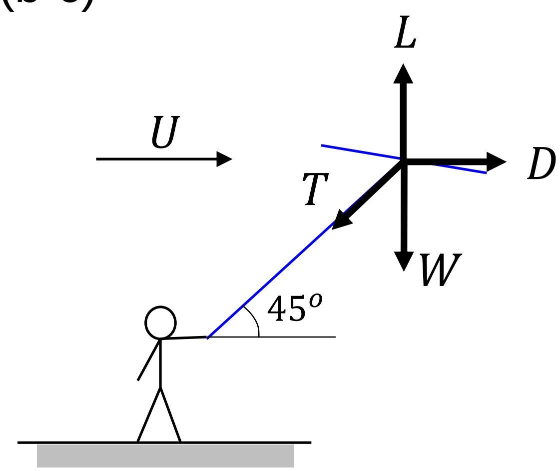
(d) Estimate the corresponding drag coefficient and lift coefficient if the planform area of the kite is 0.35 m^2. [4]

Worked example 9

Solution

(a) If the kite is parallel to the air stream direction, being the kite a flat plate, no lift is generated. Therefore, there is no force opposing gravity and the kite would fall.

(b-c)



$$L = W + T\cos 45^o = mg + T\cos 45^o$$
$$= 0.05\ kg \cdot 9.81\,\frac{m}{s^2} + 50\ N \cdot \cos 45^o$$
$$= 0.49\ N + 35.36\ N = 35.85\ N$$

$$D = T\sin 45^o = T\sin 45^o = 35.36\ N$$

(d) $C_D = \dfrac{D}{\frac{1}{2}\rho U^2 A_p},\ C_L = \dfrac{L}{\frac{1}{2}\rho U^2 A_p}$ $\qquad U = 15\,\dfrac{km}{h} = 4.17\,\dfrac{m}{s}$

$$C_D = \frac{35.36\ N}{\frac{1}{2}\cdot 1.2\,\frac{kg}{m^3}\cdot\left(4.17\,\frac{m}{s}\right)^2\cdot 0.35\ m^2} = 9.68$$

$$C_L = \frac{35.85\ N}{\frac{1}{2}\cdot 1.2\,\frac{kg}{m^3}\cdot\left(4.17\,\frac{m}{s}\right)^2\cdot 0.35\ m^2} = 9.82$$

Worked example 10

Exam paper 2019/20 – Fluids, long question

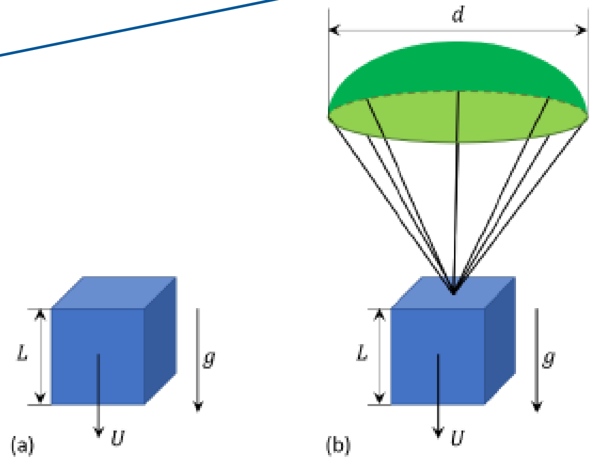
13. A cube-shaped box of side $L = 30$ cm and mass $m = 20$ kg is dropped by an airplane from an altitude of 1,000 m. For the surrounding air, you can assume a constant temperature of $T = 273$ K, density $\rho = 1.29$ kg/m³, dynamic viscosity $\mu = 1.71 \times 10^{-5}$ kg/m s. Other parameters have standard values unless stated otherwise.

(a) Name the two forces acting on the box at free-fall. [2]

(b) The box falls as indicated in Fig. Q13(a). Calculate the terminal free-fall speed of the box (terminal: when all the forces acting on the box are in equilibrium). You can use the data in Table Q13. [6]

(c) In order to reduce the speed of the box, a parachute is attached to it, as depicted in Fig. 13(b), with d being the diameter of the parachute. Calculate what this diameter should be, in order to limit the terminal speed of the box to $U = 10$ m/s. The weight of the parachute is negligible. You can assume that the box does not disturb the flow of air impacting the parachute. You can use the data in Table T13 (on next page). [5]

Slide 26



Worked example 10

Solution

(a) Drag D and gravitational force G

(b) Free-fall velocity is the speed at which $D = G$

$$D = \frac{1}{2} C_D \rho U^2 L^2, \qquad G = mg$$

Slide 26: $C_D = 1.07 \; (Re \geq 10^4)$

$$\Longrightarrow U = \sqrt{\frac{2mg}{C_D \rho L^2}} = \sqrt{\frac{2 \cdot 20\,kg \cdot 9.81\; m/s^2}{1.07 \cdot 1.29\; kg/m^3 \cdot (0.3\,m)^2}} = 56.2\,\frac{m}{s}$$

$$Re = \frac{\rho U L}{\mu} = \frac{1.29\; kg/m^3 \cdot 56.2\; m/s \cdot 0.3\,m}{1.71 \cdot 10^{-5}\; kg/(m\,s)} = 1.27 \cdot 10^6 \geq 10^4 \; OK.$$

c) With the parachute: $\frac{1}{2} C_{D,box} \rho U^2 L^2 + \frac{1}{2} C_{D,par} \rho U^2 \frac{\pi d^2}{4} = mg,$

$$d = \sqrt{\frac{8\left(mg - \frac{1}{2} C_{D,box} \rho U^2 L^2\right)}{C_{D,par} \rho U^2 \pi}}$$

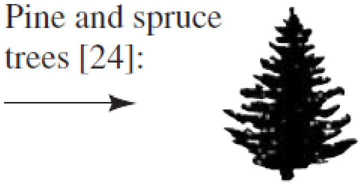
$$= \sqrt{\frac{8(20\,kg \cdot 9.81\; m/s^2 - 0.5 \cdot 1.07 \cdot 1.29\; kg/m^3 \cdot (10\; m/s)^2 \cdot (0.3\,m)^2)}{1.2 \cdot 1.29\; kg/m^3 \cdot (10\; m/s)^2 \pi}} = 1.79\,m.$$

Worked example 11

Estimate the maximum bending moment at the base of a single spruce tree in a wide open field at a wind speed of 60 mph. The tree is 3 m tall and can be treated as a cone (triangular cross-section) of base diameter 2 m and height 3 m. Consider air: ρ=1.2 kg/m^3, μ=1.8·10^{-5} Pa·s. (solution: 1029 N·m)

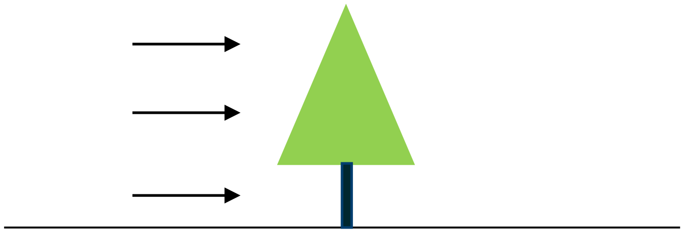
From F. White book: for $Re = \rho U t/\mu \geq 10^4$

Pine and spruce trees [24]:



U, m/s:	10	20	30	40
C_D:	1.2 ± 0.2	1.0 ± 0.2	0.7 ± 0.2	0.5 ± 0.2

(b) What would the bending moment be if the tree has a smooth trunk of diameter 0.4 m and length 1.2 m? (solution: 2301.2 N·m)



Worked example 11

Solution

We start off by evaluating the Reynolds number to make sure that we are in the regime covered by the table.

$$U = 60\, mph = 26.8\, m/s$$

Characteristic length for the Reynolds number: tree height $t = 3m$.

$$Re = \frac{\rho U t}{\mu} = 5.36 \cdot 10^6 > 10^4 \;\; \boldsymbol{OK}$$

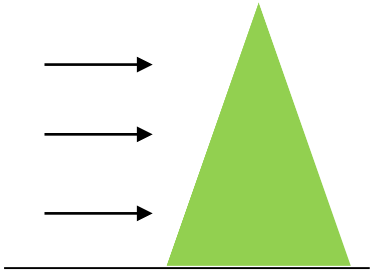
Value of C_D? From the table, $U = 20\frac{m}{s} \Longrightarrow C_D = 1, U = 30\frac{m}{s} \Longrightarrow C_D = 0.7$

We do a linear interpolation to obtain C_D at $\mathrm{U} = 26.8\, m/s$:

$$C_D = 1 + \frac{0.7 - 1}{30 - 20}(26.8 - 20) = 0.796$$

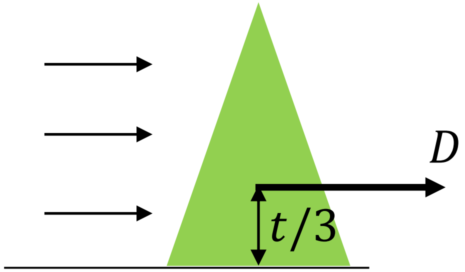
Force on the tree: $D = C_D\left(\frac{1}{2}\rho U^2\right)A = 1029\, N$

A: frontal area, $A = \frac{1}{2} d\, t = 3m^2$



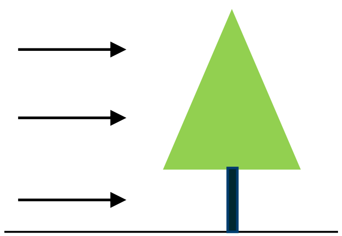
Worked example 11

To obtain the bending moment, we need to know where the force acts. The frontal area of the cone is a triangle, if we assume that the force is evenly distributed across the frontal area, the force acts at the centroid of the triangle, which is located at 1/3 of its height. Therefore, the bending moment is:



$$M = D \cdot \frac{t}{3} = 1029\ N \cdot m$$

(b) We have now to consider the presence of the trunk. This can be modeled as a 2D cylinder without considering end effects, because both ends of the cylinder at attached to something (the soil or the tree). We first check Re:



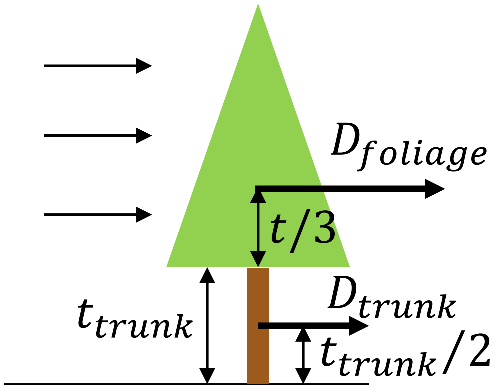
$$Re = \frac{\rho U d_{trunk}}{\mu} = 7.14 \cdot 10^5 = 10^{5.9}$$

Looking at the chart for flow past a cyilnder, slide 16 or 21, the flow can be regarded as turbulent, and we take $C_{D,trunk} = 0.3.$

Worked example 11

Therefore, the drag force exerted on the trunk is ($d_{trunk} = 0.4\ m, t_{trunk} = 1.2\ m$) :

$$D_{trunk} = C_{D,trunk}\left(\frac{1}{2}\rho U^2\right) d_{trunk} t_{trunk} = 62.1\ N$$



Now, the total bending moment will include that due to drag on the foliage, which we need to recalculate because the distance from the tree base now includes the trunk, and that due to the trunk:

$$M_{foliage} = D_{foliage} \cdot \left(\frac{t}{3} + t_{trunk}\right) = 1029 \cdot (1 + 1.2) = 2264\ N \cdot m$$

$$M_{trunk} = D_{trunk} \cdot \frac{t_{trunk}}{2} = 62 \cdot 0.6 = 37.2\ N \cdot m$$

Thus, the total makes: $M_{foliage} + M_{trunk} = 2301.2\ N \cdot m$

Worked example 12

Exam paper 2018/19 – Fluids, long question

An airplane cruises at 950 km/h at an altitude of 10,000 m (density, $\rho = 0.413\ kg/m^3$). At mid-cruise the total weight of the airplane is 250,000 kg. The fuel in the tanks of the airplane represents 33% of its total weight. All the lift applied to the airplane is generated by its wings, which have a total area of 325 m^2.

(a) Calculate the lift force generated by the wings. [2]

(b) Calculate the lift coefficient of the airplane. [3]

(c) Before touchdown, the pilot deploys three identical landing gears shown schematically in Figure Q13, one for the nose and one for each wing. At that moment the airplane is flying at 400km/h. The drag on each of the wheels is 406 N. Calculate the drag generated by the long cylinder representing the strut and hence the drag force due to the three landing gears. **Given** drag coefficient of a cylinder in the flow, which is turbulent, is Cd=0.3, and air density is 1.22 kg/m^3. [5]

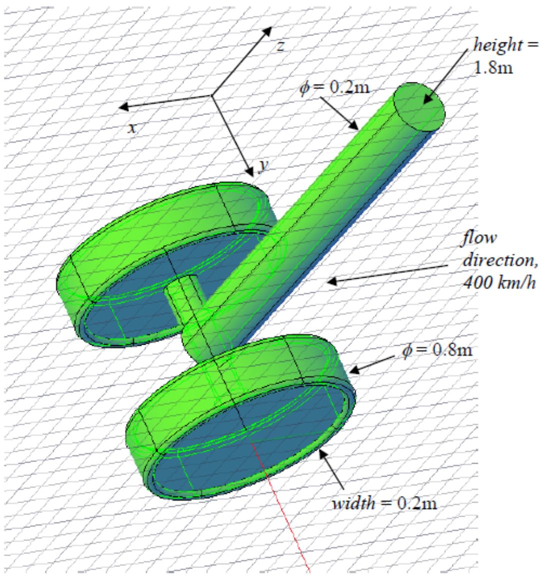


Figure Q13.

Worked example 12

Solution

(a) At cruise conditions, weight and lift must balance and therefore:

$$L = W = mg = 250000\ kg \cdot 9.81 \frac{m}{s^2} = 2452500\ N$$

(b) Lift coefficient: $C_L = \frac{L}{\frac{1}{2}\rho U^2 A_p} = \frac{2452500\ N}{\frac{1}{2} \cdot 0.413 \frac{kg}{m^3} \cdot \left(264 \frac{m}{s}\right)^2 \cdot 325\ m^2} = 0.524$

(c) $D_{total} = 6D_{wheel} + 3D_{cylinder}$

$$D_{cylinder} = C_D \left(\frac{1}{2}\rho U^2\right) A = 0.3 \left(\frac{1}{2} \cdot 1.22 \frac{kg}{m^3} \cdot \left(111 \frac{m}{s}\right)^2\right) \cdot 1.8\ m \cdot 0.2\ m = 812\ N$$

$$\Longrightarrow D_{total} = 6 \cdot 406\ N + 3 \cdot 812\ N = 4872\ N$$